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Sensitivity of the Parametrized Post Newtonian Parameter γPPN to Cosmological Models in Strong Gravitational Lensing

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Abstract

The parametrized post-Newtonian (PPN) parameter γ measures spacetime curvature per unit gravitational potential, with general relativity (GR) predicting $\gamma = 1$ exactly. Strong gravitational lensing at galactic scales offers a cosmological-scale avenue for estimating γ beyond solar system experiments; however, such estimates depend sensitively on angular diameter distances, which in turn depend on the assumed cosmological model. We perform a controlled sensitivity analysis using 40 Sloan Lens ACS (SLACS) strong lensing systems with catalogue SIE Einstein masses M_{Ein} fixed under a fiducial ΛCDM cosmology, while varying the background model across ΛCDM, wCDM, Dynamical Dark Energy (DDE), and Early Dark Energy (EDE), all adopting Planck 2018 parameters. Angular diameter distances are computed by numerically integrating the model-specific expansion function E(z), so that any variation in recovered γ_{PPN} reflects cosmological distance geometry rather than a gravitational signal. ΛCDM, wCDM, and DDE yield effectively degenerate estimates: mean $\gamma \approx 1.08 \pm 0.020$, with inter-model spread of only ~0.5–0.7%. EDE yields a systematically lower mean $\gamma = 0.903 \pm 0.019$, approximately 16.3% below ΛCDM and below the GR prediction of unity. This shift arises because EDE elevates H(z) near matter-radiation equality (z ~ 3000), compressing angular diameter distances by ~10% relative to ΛCDM; since the γ estimator scales as $D_L \times D_S / D_{LS}$, this compression propagates into a downward shift in recovered γ. The total inter-model range of ~17% substantially exceeds statistical uncertainties in targeted lensing studies, establishing cosmological model selection as a leading systematic in lensing-based γ measurements. EDE in particular introduces a distinctive geometric signature not captured by late-time dark energy parameterizations. Because M_{Ein} is fixed under ΛCDM, these findings should not be interpreted as evidence for or against GR, but as a geometric sensitivity analysis within a specific set of modeling assumptions.

Introduction

General relativity (GR), formulated by Einstein in 1915 [1], remains the most successful gravitational theory from solar system to cosmological scales [2]. One of its central predictions, the deflection of light by gravitational fields, was first confirmed by Eddington during the 1919 solar eclipse [3]. At cosmological scales, this light bending manifests as strong gravitational lensing, in which the spacetime curvature around a foreground galaxy distorts and magnifies background sources, producing observable configurations such as Einstein rings, arcs, and multiple images. Beyond their role in mapping dark matter distributions and measuring cosmic distances, strong lensing systems offer a cosmological-scale probe of gravitational physics[4, 5].

The parameterized post-Newtonian (PPN) formalism provides a systematic framework for testing gravitational theories beyond the Newtonian limit [6, 7]. Among its ten parameters, γ is the most observationally accessible: it quantifies space curvature per unit gravitational potential. GR predicts $\gamma = 1$ exactly, while scalar-tensor and other modified gravity theories predict $\gamma \neq 1$ [8]. The tightest Solar System bound comes from the Cassini radio-link experiment: $|\gamma - 1| < 2.3 \times 10^{-5}$ [9]. Extending such tests to galactic scales using strong gravitational lensing is a natural and complementary approach, probing the gravitational sector at length scales many orders of magnitude beyond the Solar System [5].

Galaxy-scale strong lensing has been employed to estimate γ at cosmological distance in several independent studies. Cao et al. [10] analyzing 118 Sloan Lens ACS (SLACS) systems with a singular isothermal sphere (SIS) mass model under a fixed ΛCDM cosmology, obtained $\gamma = 0.995 \pm 0.005$. Liu et al. [11] applied a power-law mass profile to 161 systems and found $\gamma \approx 1.07 \pm 0.07$, demontrating strong sensitivity to the assumed mass density profile. Wei et al. [12] adopted a model-independent approach based on Type Ia supernova distance reconstruction, yielding $\gamma \approx 0.995$ and explicitly mitigating the circularity between lensing analysis and cosmological assumptions. More Recently, Liu et al. [13] combined strong lensing data with gravitational-wave standard sirens to obtain a joint constraint, providing an independent distance anchor that partially decouples the γ estimate from any single cosmological model. Taken together, these studies establish that lensing-based γ estimates are sensitive not only to the lens mass model but also to the cosmological framework used to compute angular diameter distances, yet none has systematically examined how different dark energy models, particularly those motivated by the Hubble tension, propagate into the recovered γ.

A common feature across these studies is their reliance on a fixed cosmological model (typically ΛCDM) when computing angular diameter distances D_L, D_S, and D_{LS} that enter the γ estimator. However, these distances are sensitive to the assumed expansion history E(z), which differs substantially across cosmological models [14, 15]. This sensitivity is particularly pronounced for Early Dark Energy (EDE) [16, 17], a class of models proposed as a resolution to the Hubble tension [18-20], the ~5σ discrepancy between H_0 inferred from CMB measurements and from local distance ladder observations. EDE introduces a dark energy component that that become dinamically significant near matter-radiation equality ($z \sim 3000$), substantially modifying the pre-recombination expansion history and, consequently, reducing all angular diameter distances relative to the ΛCDM [21, 22]. This is in contrast to late-time dark energy models such as wCDM and Dynamical Dark Energy (DDE), which modify E(z) only at $z \lesssim$ a few and produce comparatively modest distance shift. While Cao et al. [10] and

Wei et al [12] acknowledged the role of cosmological assumptions in lensing-based γ estimates, and Liu et al. [13] partially addressed this through independent distance anchors, no study has yet performed a systematic, model-to-model comparison spanning both late-time and early-universe dark energy framework within a self-consistent lensing analysis. In particular, the degree to which EDE's distinctive modification of the early expansion history propagates into lensing-based γ estimates remains unquantified. This constitutes the specific gap that motivates the present work.

In this paper, we address this gap through a controlled cosmological sensitivity analysis. Using 40 SLACS strong lensing systems [23] with catalogue SIE Einstein lensing masses M_{Ein} fixed under a fiducial ΛCDM cosmology, we vary cosmological model across ΛCDM, wCDM, DDE [14, 15, 24], and EDE [16], and quantify the resulting variation in the recovered γ_{PPN}. By holding M_{Ein} fixed, any change between models reflects purely the change in the angular diameter distance geometry, providing a clean model-to-model comparison of the cosmological contribution to the γ systematic budget, an aspect not previously addressed in the literature. We stress that this design choice means our results should be interpreted as a geometric sensitivity study, not as an independent gravitational test: the recovered γ for non-ΛCDM models reflects the distance ratio between that model and the ΛCDM baseline used to calibrate M_{Ein}, rather than a genuine deviation from GR. This framing is intentional and, we argue, constitutes the paper's primary contribution: a quantification of how strongly and differentially cosmological model choice, especially the inclusion of EDE, propagates into lensing-based γ estimates. Section 2 presents the theoretical framework; Section 3 describes data and methodology; Sections 4 and 5 present results and discussion; Section 6 concludes.

Theory and Calculation

PPN Metric and Light Deflection

In the PPN formalism, the spacetime metric around a static, spherically symmetric mass in isotropic coordinates is, to first post-Newtonian order [6-8]:

$$ds^2 = -\left(1 - \frac{2U}{c^2}\right) c^2\, dt^2 + \left(1 + 2\gamma_{\text{ppn}} \frac{U}{c^2}\right)(dr^2 + r^2\, d\Omega^2), \tag{1}$$

where $U = GM/r$ and γ_{PPN} modifies the spatial metric component. For $\gamma_{PPN} = 1$, Eq. (1) recovers the weak-field isotropic Schwarzschild limit. Solving the null geodesic equation in this metric yields the deflection angle for a light ray at impact parameter b [7, 10, 25]:

$$\alpha = \left(\frac{(1 + \gamma_{\text{ppn}})}{2}\right) \times \left(\frac{4GM}{c^2 b}\right). \tag{2}$$

For $\gamma_{PPN} = 1$, Eq. (2) recovers the standard GR deflection $\alpha_{GR} = 4GM/(c^2 b)$.

Einstein Ring Radius and the γ Estimator

Here, β denotes the angular position of the background source as it would appear in the absence of gravitational lensing, measured from the optical axis connecting the observer and the lens centre. The term $(D_{LS}/D_S)\alpha$ represents the angular deflection of the image position relative to the true source position, scaled by the ratio of the lens-source to observer-source angular diameter distances. Perfect alignment between the observer, lens, and source ($\beta = 0$) produces a symmetric ring image, the Einstein ring, at angular radius θ_E, from which Eq. (3) follows directly. A schematic of the lensing geometry is shown in Fig.1

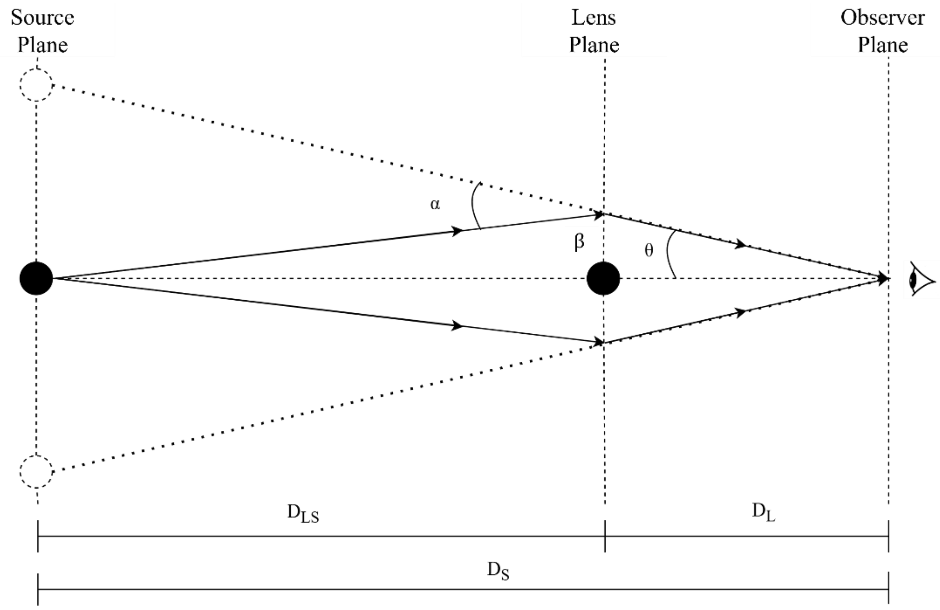

Figure 1. Schematic diagram of the gravitational lensing geometry. A background source (open circle, Source Plane) is deflected by a foreground lens galaxy (filled circle, Lens Plane) before reaching the observer (Observer Plane). The solid lines show the actual deflected light paths; dotted lines show the unperturbed ray directions. The quantity α is the deflection angle at the lens, β is the angular position of the true (unlensed) source measured from the optical axis, and θ is the observed image position angle. The angular diameter distances D_L (observer to lens), D_S (observer to source), and D_{LS} (lens to source) are indicated by the brackets below.

When source, lens, and observer are collinear, the equation $\beta = \theta - (D_{LS}/D_S)\,\alpha(\theta) = 0$ yields the Einstein ring radius θ_E [3, 26]:

$$\theta_E = \sqrt{\left(\frac{(1 + \gamma_{\text{ppn}})}{2}\right) \times (4GM / c^2) \times \left(\frac{D_{LS}}{D_L D_S}\right)}. \tag{3}$$

Solving for γ_{ppn} gives the estimator used throuout this study:

$$\gamma_{\text{ppn}} = \left[c^2\theta_E^2 \frac{D_L D_S}{2GM(< b_E)D_{LS}}\right] - 1, \tag{4}$$

where $M(<b_E)$ is the projected mass enclosed within the Einstein radius $b_E = D_L\theta_E$, and D_L, D_S, D_{LS} are angular diameter distances from observer to lens, observer to source, and lens to source. Eq. (4) shows that γ_{ppn} is sensitive to both the lensing mass and the cosmological distances [10-12].

Cosmological Models and Angular Diameter Distances

The angular diameter distance in a flat universe is [27, 28]:

$$\text{D}(z) = \left(\frac{c}{H_0}\right) \int_0^z \frac{dz'}{(1+z')E(z')}, \tag{5}$$

where $E(z) = H(z)/H_0$ is the dimensionless expansion rate. We consider four models (Table 1), all assuming flat geometry ($\Omega_k = 0$) and Planck 2018 base parameters $H_0 = 67.4$ km s^{-1} Mpc^{-1}, $\Omega_m = 0.315$ [27].

Table 1. Cosmological models and E(z) functions used in this study.

Model	E(z)	Dark energy EoS	Ref.
ΛCDM	$\sqrt{\Omega_m(1+z)^3 + \Omega_\Lambda}$	$w = -1$	[24, 27, 28]
wCDM	$\sqrt{\Omega_m(1+z)^3 + \Omega_{DE}(1+z)^{3(1+w)}}$	$w = -1.1$	[24, 29]
DDE	$\left[\Omega_m(1+z)^3 + \Omega_{DE}(1+z)^{3(1+w_0+w_a)} exp\left(-\frac{3w_a z}{1+z}\right)\right]^{\frac{1}{2}}$	$w(z) = w_0 + \frac{w_a z}{1+z}$; $w_0 = -1.0$, $w_a = 0.5$	[14, 15, 24]
EDE	$E^2(z) = \frac{\Omega_m(1+z)^3}{1-\Omega_{EDE}(z)}$	$\Omega_e = 0.05, n = 4$	[16, 17, 21]

The distance D_{LS} is computed as $D_{LS} = \left[\frac{1}{1+z_S}\right] \int \frac{dz'}{E(z')}$ from z_L to z_S. All integrals are evaluated with scipy.integrate.quad [30], with relative precision < 10^{-6}.

Experimental Method

Lensing Sample

We use catalogue Einstein masses M_{Ein} from [23], which was derived from SIE profile fitting [31] under a fiducial ΛCDM cosmology ($H_0 = 70$ km s^{-1} Mpc^{-1}, $\Omega_m = 0.3$). The SIE (Singular Isothermal Ellipsoid) profile assumes a power-law mass slope of $\gamma_{mass} = 2$; while standard for early-type lens galaxies, the inferred γ_{PPN} is sensitive to this choice, Liu et al. [11] showed that adopting a free power-law slope shifts the recovered γ by ~7–8% relative to the isothermal case, comparable in magnitude to the cosmological systematic quantified in this work. Since M_{Ein} is fixed under a fiducial ΛCDM cosmology and a fixed SIE profile, the present analysis cannot disentangle the contributions of the mass-model and cosmological-model systematics to the total γ budget; we treat this as an explicit limitation of the study. These masses are held fixed across all four cosmological models in our analysis, so that any variation in the recovered γ_{PPN} between models arises purely from differences in the angular diameter distances D_L, D_S, and D_{LS}, and not from any change in the lens mass itself.

This design choice has an important methodological consequence that must be stated explicitly: because M_{Ein} is calibrated under ΛCDM, it implicitly encodes the ΛCDM distance scale. When the cosmological model is varied, the distances change but M_{Ein} does not, so the recovered γ for any non-ΛCDM model reflects the ratio of the alternative model's distance geometry to the ΛCDM geometry used during mass calibration, not a genuine deviation from GR. Put differently, our analysis isolates the cosmological contribution to the γ systematic budget in a controlled way, but cannot separate the gravitational signal from the cosmological distance geometry without a self-consistent joint fit. We therefore characterize this study as a geometric sensitivity analysis rather than a gravitational test. This approach is deliberately chosen because it provides a clean, model-to-model comparison of how the background cosmology propagates into lensing-based γ estimates, an aspect not previously addressed for EDE in the literature. We note that relaxing the assumption of a fixed ΛCDM-calibrated M_{Ein} would produce small corrections for wCDM and DDE, given that these models deviate from ΛCDM by only ~1–2% in angular diameter distance at the redshifts of our sample ($z_L \sim 0.1 - 0.5, z_S \sim 0.5 - 1.0$). For EDE, where the distance deviation reaches ~10%, the correction from re-fitting M_{Ein} self-consistently could be non-negligible. Quantifying this correction requires re-fitting the SIE mass model directly to the SLACS imaging data under each cosmological model, which is beyond the scope of the present analysis but constitutes a natural and important extension of this work.

As a practical advantage, fixing M_{Ein} to the imaging-derived SIE mass also avoids the large intrinsic scatter ($\sigma_\gamma \sim 0.44$) that arises when velocity-dispersion-based masses are used, since spectroscopic and imaging-derived masses are not mutually self-consistent [13, 32].

Estimation Procedure

For each of the 40 lensing system and each of the four cosmological models, the estimation proceeds as follows. The lens redshifts z_L and source redshifts z_S are taken directly from the SLACS spectroscopic catalogue of [23] and are held fixed across all four cosmological models; only the expansion function E(z) and hence the angular diameter distances varies between models. First, the angular diameter distance D_L, D_S, D_{LS} are computed by numerically integrating Eq. (5) with the model-specific E(z) using scipy.integrate.quad [33]; unit conversions are handled via Astropy [33]. Second the Einstein radius impact parameter is calculated as $b_E = D_L\theta_E$, where θ_E is converted from arcseconds to radians. Third, γ_{PPN} is evaluated via Eq. (4) using fixed M_{Ein} and model-specific distances. Finally, the mean, standard deviation, and median of γ are computed over all 40 systems. All 40 systems yield positive, physically meaningful γ values; no additional exclusions are applied.

Result and Discussion

Angular Diameter Distances

Table 2 presents mean angular diameter distances over the 40-system sample. The ΛCDM, wCDM, and DDE models agree within ~2% across all three distances, consistent with the fact that these models differ only in their late-time equation of state ($z \lesssim$ a few) and thus produce negligible differences in the cumulative comoving distance out to lens and source redshifts ($z_L \sim 0.1$–0.5, $z_S \sim 0.5$–1.0). The EDE model, by contrast, reduces D_L and D_S by ~9–10% relative to ΛCDM, a consequence of its elevated expansion rate H(z) near matter-radiation equality (z ~ 3000), which compresses the comoving distance scale throughout cosmic history. Despite this compression, the ratio D_S/D_{LS} remains nearly identical across all models (1.732–1.749), a consequence of the flat-universe distance sum rule. This geometry means that the dominant driver of inter-model differences in γ is the absolute distance scale entering the numerator of the γ estimator (Eq. 4), specifically the product $D_L \times D_S / D_{LS}$, not any differential shift in the distance ratio alone.

Table 2. Mean angular diameter distances over 40 lensing systems for each cosmological model.

Model	<D_L> (Mpc)	<D_S> (Mpc)	<D_{LS}> (Mpc)	<D_S/D_{LS}>	$\Delta D/D\Lambda$
ΛCDM	606.66	1367.97	911.44	1.7406	*ref.*
wCDM	611.44	1389.75	929.56	1.7328	+1.6%
DDE	605.18	1352.00	896.64	1.7492	−0.2%
EDE	552.76	1236.83	820.92	1.7481	−10.0%

Estimated γ_{ppn}

Table 3 presents the main results: the mean, standard deviation, median, and deviation from the GR prediction (γ = 1) for each cosmological model. Fig. 2 (described in next Section) shows the distribution of individual γ values.

Table 3. Summary of γ_{PPN} estimates for each cosmological model (N = 40 systems).

Model	Mean γ	Std Dev	Median γ	Δγ vs GR	Δγ vs ΛCDM
ΛCDM	1.079	0.020	1.077	+7.9%	*ref.*
wCDM	1.086	0.019	1.084	+8.6%	+0.7%
DDE	1.084	0.020	1.082	+8.4%	+0.5%
EDE	0.903	0.019	0.901	−9.7%	−16.3%

Three results stand out from Table 3. First, ΛCDM, wCDM, and DDE are effectively degenerate in their γ estimates: their means span only Δγ = 0.007, well within the per-system scatter σ ~ 0.020. This degeneracy is expected given that these models produce nearly identical distance geometries at the lens and source redshifts of this sample ($z_L \lesssim 0.5$, $z_S \lesssim 1.0$). Second, all three late-time models yield γ ≈ 1.08 ± 0.020, approximately 8% above the GR prediction. This is consistent with the power-law mass profile result of Liu et al. [11] (γ ≈ 1.07 ± 0.07), suggesting that the ~8% offset may partially reflect a systematic associated with the SIE mass model adopted here rather than a genuine gravitational deviation, a point we return to in Section 5. Third, EDE yields γ = 0.903 ± 0.019, approximately 16.3% below the ΛCDM value and below the GR prediction of unity. The uniformly small scatter across all 40 systems ($\sigma_{EDE} \approx \sigma_{\Lambda CDM} \approx 0.020$) confirms that this is a coherent systematic shift, not an outlier-driven effect.

The ~17% inter-model range in mean γ across models substantially exceeds the statistical precision achieved in targeted studies (σ ~ 0.005 in Cao et al. [10]; σ ~ 0.009 in Wei et al. [12]). This comparison reveals a hierarchy of systematics: the cosmological model uncertainty quantified here (~17%) exceeds the statistical floor of precision lensing studies by more than an order of magnitude, and is comparable in magnitude to the systematic from mass model choice. Liu et al. [11] showed that switching from SIS to a power-law profile shifts γ by ~7–8%, and Leaf & Melia [32] further demonstrated sensitivity to model selection criteria. Crucially, these two sources of systematic (mass model and cosmological model) are not independent: fixing M_{Ein} under a fiducial ΛCDM cosmology, as done here and in [10, 11], conflates both effects. This motivates three complementary strategies for future work : (a) adopting a cosmological parameters jointly constrained by CMB+BAO+SN data [27]; (b) jointly fitting γ and cosmological parameters within full Bayesian framework; or (c) using model-independent distances reconstructed from Type Ia supernovae [12] or time-delay cosmography [34]. Gravitational wave standard sirens [13, 36] offer an additional independent distance anchor that is insensitive to both lens mass assumptions and dark energy parameterization.

Our EDE result does not imply GR violation. Rather, it demonstrates that a lensing-based analysis performed under an assumed ΛCDM background would misattribute EDE's modified distance geometry as a departure from GR, recovering $\gamma \sim 0.90$ even if gravity is exactly Einsteinian. This is a direct manifestation of the degeneracy between γ_{PPN} and the cosmological background, previously noted in the literature [10, 12] but not yet quantified for EDE specifically. The Hubble tension therefore introduces a concrete ambiguity in lensing-based gravity tests: if the true cosmology is EDE-like, then all ΛCDM-based γ analyses in the literature are systematically biased toward $\gamma > 1$, and the apparent consistency with GR in studies such as [10, 12] could partly reflect a cancellation of the EDE distance bias by other modeling choices. We emphasize that resolving this degeneracy requires data beyond strong lensing alone; combining lensing γ estimates with CMB power spectra, BAO measurements, and gravitational wave standard sirens [13, 36] would allow simultaneous constraints on both γ and the EDE fraction.

Robustness and Redshift Dependence

The distributions of individual γ values across the 40 systems are approximately Gaussian for all four models, with comparable widths ($\sigma \sim 0.019$–0.020). The ΛCDM, wCDM, and DDE distributions are nearly indistinguishable, centered near $\gamma \approx 1.08$. The EDE distribution is uniformly shifted to $\gamma \approx 0.90$, with no significant broadening relative to the ΛCDM distribution ($\sigma_{EDE}/\sigma_{\Lambda CDM} \approx 0.95$). This comparable width is informative: if EDE's effect were driven by a subset of systems at particular lens configurations or redshifts, one would expect asymmetric tails or elevated dispersion in the EDE distribution. The absence of such features confirms that EDE introduces a coherent, system-independent shift in the distance geometry, consistent with the fact that EDE modifies the overall expansion history rather than any lens-specific property.

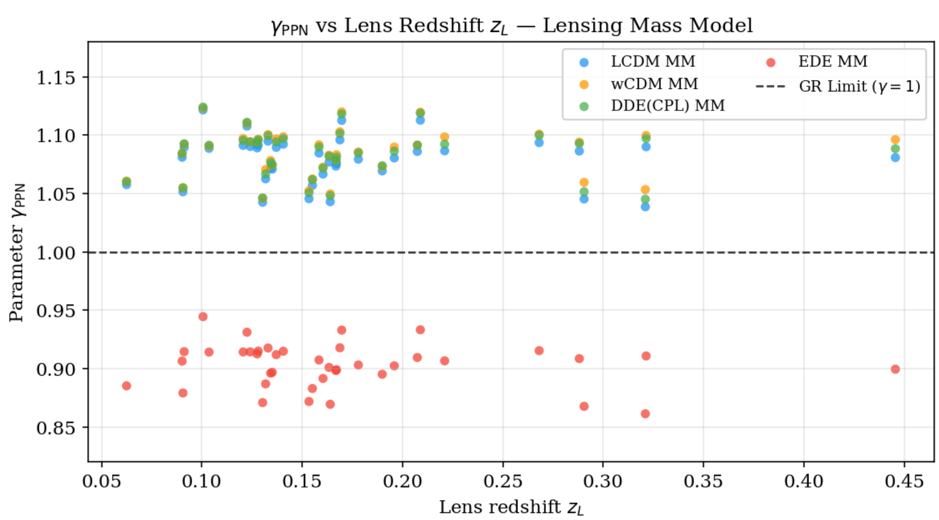

Figure 2. Estimates of γ_{PPN} from the lensing mass model as a function of lens redshift z_L. There is no significant redshift trend, indicating the absence of selection bias toward systems at specific redshifts.

We further examine γ as a function of lens redshift z_L (range 0.06–0.45) to assess whether the EDE shift is uniform across the sample or concecrated in particular redshift bins. For all four models, there is no statistically significant correlation between γ and z_L (Pearson $|r| < 0.05$), and the slope of a linear fit consistent with zero. This has two implications. First, it confirms that the sample does not suffer from a redshift-dependent selection bias that could spuriously amplify or suppress the EDE effect at any particular epoch. Second, and more importantly for the robustness of the EDE result, it confirms that the ~16% shift in mean γ under EDE is distributed uniformly across the full redshift range of the sample, not driven by a subset of low-z or high-z systems where EDE's distance modification might be disproportionately large.

Several limitations of this analysis warrant explicit acknowledgment. First, the circularity inherent in our design, fixing M_{Ein} under a fiducial ΛCDM calibration while varying the cosmological model, means that the recovered γ for non-ΛCDM models reflects the ratio of distance geometries between the alternative and ΛCDM frameworks, not a genuinely model-independent gravitational measurement. This is the same limitation present in all analyses that adopt catalogue lensing masses without re-deriving them under each tested cosmology. A self-consistent treatment would require re-fitting M_{Ein} under each cosmological model simultaneously, an approach that demands full access to the imaging data and is beyond the scope of the present sensitivity study. Second, the γ estimator used here (Eq. 4) does not account for the contribution of the line-of-sight mass structure or the lens environment, which can introduce additional scatter and systematic offsets in individual γ estimates [10, 13]. Third, the SLACS sample spans a relatively narrow redshift range ($z_L \sim 0.06$–0.45), which limits sensitivity to cosmological models whose distance deviations from ΛCDM grow more pronounced at $z > 0.5$. Taken together, these limitations reinforce the interpretation of this work as a geometric sensitivity analysis within a specific set of assumptions, rather than a definitive test of gravity at galactic scales.

Conclusion

We have carried out a controlled cosmological sensitivity analysis of the lensing-based γ_{PPN} estimator, using 40 SLACS strong lensing systems [23] with fixed catalogue SIE masses M_{Ein} and systematically varying the background cosmological model across ΛCDM, wCDM, DDE, and EDE. The principal findings are as follow. Late-time dark energy models (ΛCDM, wCDM, DDE) yield effectively degenerate γ estimates: mean $\gamma \approx 1.08$ ($\sigma \sim 0.020$) with inter-model spread of only ~0.5–0.7%, consistent with Liu et al. [11] and reflecting the near-identical angular diameter distance geometry these models produce at $z \lesssim 1$. EDE yields a systematically lower mean $\gamma = 0.903$ ($\sigma \sim 0.019$), approximately 16% below ΛCDM and crossing below the GR prediction, driven by its ~10% reduction of angular diameter distances through modification of the pre-recombination expansion history. This EDE shift is robust : the distribution width is comparable to ΛCDM ($\sigma_{EDE}/\sigma_{\Lambda CDM} \approx 0.95$), no significant γ–$z_L$ correlation is found (Pearson $|r| < 0.05$ for all models), and the shift uniform across the full lens redshift range $z_L \sim 0.06$–0.45. The total inter-model span of ~17% in mean γ subtantially exceeds the statistical precision of targeted lensing studies and is comparable magnitude to the systematic from mass model choice, establishing cosmological model selection as a leading source of systematic uncertainty in lensing based γ measurement.

These results should be interpreted within the limitations of the adopted methodology. Because M_{Ein} is calibrated under a fiducial ΛCDM cosmology and held fixed across all models, the recovered γ for non-ΛCDM models reflects the ratio of distance geometries between the alternative framework and ΛCDM, rather than an independent gravitational measurement. Our results therefore do not constitute a test of GR at galactic scales, but rather a quantification of how sensitively the lensing-based γ estimator responds to changes in the cosmological background, a distinction we consider important to state explicitly given the ongoing discussion of EDE as a resolution to the Hubble tension. In particular, our EDE result ($\gamma \approx 0.90$) illustrates that a ΛCDM-based lensing analysis would misidentify EDE's modified distance geometry as a sub-unity γ, even in a universe where GR holds exactly. This degeneracy between γ_{PPN} and cosmological parameters cannot be broken by lensing data alone.

Looking forward, this work motivates several directions for more rigorous analyses. The most immediate priority is a self-consistent treatment in which M_{Ein} is re-derived under each cosmological model simultaneously, rather than fixed under a fiducial ΛCDM, this would remove the circularity inherent in the present analysis and yield a genuinely model-independent gravitational test. Combining lensing-based γ estimates with independent distance anchors from gravitational wave standard sirens [13] or time-delay cosmography [34] offers a complementary route to breaking the degeneracy between γ and the cosmological background without requiring simultaneous mass re-fitting. Larger lensing samples from upcoming surveys such as Euclid [36] and JWST-based catalogues will also enable joint constraints on γ_{PPN} and the EDE fraction, with improved statistical power to detect or exclude the ~16% cosmological systematic identified here. Finally, extending this sensitivity analysis to higher-redshift lens systems ($z_L > 0.5$), where the distance deviation between EDE and late-time models grows more pronounced, would sharpen the discriminating power of the lensing geometry approach.

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