Indonesian Physical Review

Volume 08 Issue 02, May 2025

P-ISSN: 2615-1278, E-ISSN: 2614-7904

Hawking Temperature in Schwarzschild Black Holes with Quintessence Dark Energy

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Article Info

Article info:

Received: 23-12-2024

Revised: 21-05-2025

Accepted: 22-05-2025

Keywords:

Hawking temperature; Black hole; Schwarzschild de-Sitter; quintessence

How To Cite:

A. Widiyani, A. N. Sari, I. N. P. Permanasari, E. Satria, A. S. K. Pohan, W. A. P. Kesuma, I. Pardede, and A. Suroso, "Hawking Temperature in Schwarzschild Black Holes with Quintessence Dark Energy", Indonesian Physical Review, vol. 8, no. 2, p 553-568, 2025.

DOI:

https://doi.org/10.29303/ipr.v8i2.454.

Abstract

Black holes are thermodynamic objects that emit Hawking radiation near the event horizon of a black hole according to the theory of quantum gravity in curved space-time. This radiation is manifested as the temperature of a black hole, known as the Hawking temperature. According to black hole thermodynamics, the black hole horizon area corresponds to the entropy. The increase in the horizon area is predicted due to the influence of dark energy, which can push the horizon of the black hole away from its center, thus significantly affecting the radiation of the black hole. Here, we investigate the Hawking temperature of the Schwarzschild black hole under the effect of quintessence dark energy. The results show that the increase in quintessence reduces the horizon radius of the black hole and lowers its Hawking temperature, highlighting the direct relationship between dark energy and black hole dynamics.

Introduction

Black holes are celestial objects with a region of spacetime that has such strong gravity that nothing, not even light, can escape from it. According to the theory of Albert Einstein's general relativity, an object with sufficient mass can deform in spacetime and form a black hole [1]. For many years, the black hole region was theoretically described as a space-time bounded by an event horizon [2]. This phenomenon is closely related to the universe's accelerated expansion, which is currently assumed to be driven by dark energy [3-7]. Consequently, the black hole horizon expands, influencing radiation emission from the black hole [8].

The use of thermodynamic concepts has opened a useful approach in understanding various processes related to horizons, such as the evaporation of black holes. The concept of black hole thermodynamics began to develop in the early 1970s based on the argument that the entropy of a black hole is directly proportional to the horizon area [9]. Stephen Hawking in 1975 demonstrated that black holes emit radiation near their event horizons by integrating quantum mechanics and general relativity concepts [10]. The horizon area increases as more matter falls into the black hole. This event horizon area corresponds to entropy, which represents the heat energy emitted as Hawking radiation [11-16]. Since the event horizon possesses entropy, it relates to temperature through the first law of thermodynamics [12-14]. The temperature measured near the black hole's event horizon is referred to as the Hawking temperature. The rate of black hole radiation emission depends on mass, angular momentum, and charge [17].

In four-dimensional (4D) spacetime with dark energy, black holes have been modelled using the Schwarzschild-de Sitter (SdS) black hole model [18-19]. This model combines the Schwarzschild black hole, describing small-scale phenomena over brief periods, with the Friedmann-Robertson-Walker (FRW) cosmological model, which represents larger-scale, long-term cosmic evolution. The SdS black hole features two event horizons, with the temperatures derived from their surface gravity [20]. In 2017, Pappas and Kanti calculated the temperature of the SdS black hole using the cosmological constant (Λ) as a representation of dark energy [12]. While Λ is the simplest and most widely used model to describe dark energy, it faces challenges such as the fine-tuning and coincidence problems [4-5, 21]. As an alternative, quintessence was introduced by Ratra and Peebles in 1988 as a dynamic scalar field model whose energy density evolves with time. This evolution provides a natural explanation for the onset of cosmic acceleration and avoids the static nature of Λ. Compared to phantom energy models ($\omega < -1$), which often predicts unphysical scenarios like future singularities, maintains theoretical consistency within Einstein's general theory of relativity and quantum field theory. Because of its dynamic character and physical motivation, quintessence presents a promising and viable theoretical model for studying dark energy's influence in both cosmology and black hole physics [22].

In this study, the Hawking temperatures are analysed in the spacetime of Schwarzschild black holes influenced by quintessence, as formulated by Kiselev [23-24]. Kiselev was the first to solve the Einstein field equations using a generalized Schwarzschild metric by incorporating a quintessence field as the source of dark energy, which is believed to affect the spacetime structure surrounding black holes. While Fernando (2014) analyses the thermodynamic behaviour of Schwarzschild black holes surrounded by both quintessence and a cosmological constant using a fixed equation of state $\omega = -2/3$ [25], our study contributes a different perspective. Specifically, we focus on three distinct temperature quantities: the unnormalized Hawking temperature T_0, the normalized temperature at the black hole horizon T_{BH}, and the temperature associated with the cosmological horizon T_c . Detailed analysis is conducted on each temperature quantity with respect to variations in the quintessence normalization parameter (q) and the black hole mass (m). Through comparative graphical analysis, we explore how the presence of quintessence leads to thermal asymmetry between horizons. These aspects offer a novel thermodynamic interpretation of black hole behavior in the presence of dynamic dark energy.

In this paper, the discussion begins with the background of this research. Then the second part contains the theoretical basis and calculations for the Schwarzschild solution in the quintessence model. Afterward, we calculated the Hawking temperature (T_0) in the third part. Subsequently, normalization was performed by modifying the Schwarzschild radius (r_0) to obtain the black hole temperature or Hawking temperature with normalization (T_{BH}). In the fourth part, we describe the results and discussion of the calculation and analysis of the Hawking temperature of the Schwarzschild black hole obtained. We created temperature graphs using Wolfram Mathematica software. The last part of the paper is closed with a conclusion. All equations and calculations in this work are expressed in natural units ($G \approx c \approx \hbar \approx k_B \approx 1$). In this unit system, physical parameters, including mass, length, and temperature, are represented in consistent dimensionless form.

The Schwarzschild Blackhole with Quintessence

In the first few seconds of the formation of the universe, which was the beginning of the formation of primordial black holes, dark energy was not yet a dominant component in the cosmic energy. While it may have initially seemed that black holes were unrelated to dark energy, this notion is conceptually inaccurate. Dark energy interacts with gravity by contributing to the energy-momentum tensor through the energy density and the negative pressure ($p_q = \omega_q \rho_q$, with $\omega_q < 0$). The negative pressure gives rise to a repulsive gravitational force, contributing to the universe's faster rate of expansion and causes everything in the universe to move away, as supported by solutions to Einstein's field equations in cosmology [27-28]. Two major models have been proposed to describe dark energy, i.e. the cosmological constant (Λ) which representing the energy density of a constant vacuum and the models of a dynamic scalar field such as quintessence, phantom, k-essence, chameleon, tachyon, and dilaton [4].

Quintessence is one of the dark energy candidates that combines scalar fields with gravity. The influence of quintessence has been studied from various perspectives, such as the shift in gravitational frequencies and the bending of light [28]. If quintessence exists throughout the universe, then quintessence can also exist around black holes. In this research, we have studied the exact solution of static spherical symmetry with a black hole surrounded by quintessence [23]. Let us start by considering the spherically symmetric spacetime:

$$ds^2 = -e^{\alpha} dt^2 + e^{\beta} dr^2 + r^2 d\theta^2 + \sin^2 \theta r^2 d\phi^2 , \tag{1}$$

where α, β denotes a function with respect to the radial distance (r) [29]. The relationship between spacetime geometry and the energy-momentum distribution of quintessence is expressed in the Einstein's field equations

$$G^{\mu}_{\nu} = g^{\mu\nu}\left(R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \right) = \frac{8\pi G}{c^2} T^{\mu(q)}_{\nu} , \tag{2}$$

where G^{μ}_{ν}, $g_{\mu\nu}$, $R_{\mu\nu}$, R, G, c, and $T^{\mu(q)}_{\nu}$ are the Einstein tensor, the metric tensor, the Ricci tensor, Ricci scalar, the Einstein gravitational constant, the speed of light, and the energy-momentum tensor for quintessence, respectively. By using the natural units, $4\pi G \approx c \approx \hbar \approx 1$, the Einstein's equations are obtained as follows

$$2T_t^t = -e^{-\beta}\left(\frac{1}{r^2} - \frac{\beta'}{r}\right) + \frac{1}{r^2}, \tag{3}$$

$$2T_r^r = -e^{-\beta}\left(\frac{1}{r^2} + \frac{\alpha'}{r}\right) + \frac{1}{r^2}, \tag{4}$$

$$2T_\theta^\theta = 2T_\phi^\phi = -\frac{1}{2}e^{-\beta}\left(\alpha'' + \frac{\alpha'^2}{2} + \frac{\alpha' - \beta'}{r} - \frac{\alpha'\beta'}{2}\right), \tag{5}$$

where the prime $(\ldots')$ represents the derivative of the radial distance r (example: $\alpha' \equiv d\alpha/dr$). The energy-momentum tensor for quintessence [23, 28-29] is given as:

$$T_t^t = T_r^r = \rho_q, \tag{6}$$

$$T_\theta^\theta = T_\phi^\phi = -\tfrac{1}{2}\rho_q\left(3\omega_q + 1\right), \tag{7}$$

where ω_q and ρ_q are the energy density and the equation of state parameter for quintessence, which represents the ratio of pressure and energy density of quintessence, $\omega_q = p_q/\rho_q$.

If we define the principle of addition and linearity of the equation (3-4) by placing the relationship between the metric components, we obtain

$$T_t^t = T_r^r \quad \Rightarrow \quad \beta + \alpha = 0\,. \tag{8}$$

Without losing its generality because the static coordinate system is set by the gauge above $\beta + \alpha = \mathrm{const} = 0$, then the constant can be eliminated by performing an appropriate time rescaling. Then, λ is defined as a linear differential equation in *f*, which is written as follows

$$\beta = -\ln\left(1 + f\right), \tag{9}$$

with f is a linear differential function [23]. Substitute eq. (9) to Einstein's field equations (3-5), so that

$$T_t^t = T_r^r = -\frac{1}{2r^2}\left(f + rf'\right) \tag{10}$$

$$T_\theta^\theta = T_\phi^\phi = -\frac{1}{4r}\left(2f + rf''\right) \tag{11}$$

From (6-7) and (10-11), the result is

$$r^2 f'' + 3\left(\omega_q + 1\right)rf' + \left(3\omega_q + 1\right) f = 0\,. \tag{12}$$

It has two solutions in the form

$$f_q = \frac{q}{r^{3\omega_q + 1}}, \tag{13}$$

$$f_{BH} = -\frac{2m}{r}, \tag{14}$$

with q is the factor of normalization dependent on the energy density of quintessence and m is the mass of black hole. The function f_{BH} denotes the usual Schwarzschild solution for a point-like black hole and this corresponds to a special choice of dust matter with $\omega_q = 0$ in f_q that will give the quintessence energy density (ρ_q) at $r \neq 0$. If taken $\rho_q > 0$, then

$$\rho_q = -\frac{q}{2}\frac{3\omega_q}{r^{3(\omega_q+1)}}. \tag{15}$$

From equations (6) and (7), the resulting trace of the tensor of energy-momentum is given by

$$T = T_\mu^\mu = g^{\mu\nu}T_{\mu\nu} = g^{tt}T_{tt} + g^{rr}T_{rr} + g^{\theta\theta}T_{\theta\theta} + g^{\phi\phi}T_{\phi\phi}.$$

$$T = \rho_q\left(1-3\omega_q\right) \tag{16}$$

The expression for the Ricci scalar is derived from the trace of Einstein's equations (eq. 2), then the Ricci scalar is in the form of

$$R = g^{\mu\nu}R_{\mu\nu} = 2T_\mu^\mu = 3q\omega_q\frac{\left(1-3\omega_q\right)}{r^{3(\omega_q+1)}}. \tag{17}$$

As shown in eq. (17), it exhibits a singularity at r = 0 when $\omega_q \neq \left\{0, \frac{1}{3}, -1\right\}$ [23]. The equation shows that the energy density of quintessence directly affects the curvature of space-time. This divergence in the Ricci scalar indicates a true physical curvature singularity, similar in nature to the central singularity found in a Schwarzschild black hole, and not simply a mathematical construct or a coordinate-dependent singularity. Since the Ricci scalar is an invariant geometric quantity, this divergence signifies that the energy density of the quintessence becomes infinite at the origin. However, because this singularity is shielded within the event horizon, it remains unobservable to external observers, in accordance with the cosmic censorship conjecture [30].

In this study, we adopt a static spherically symmetric solution as proposed by Kiselev (2003), which approximates the quintessence effects of the region surrounding a black hole. Although quintessence is represented by a scalar field that evolves over time, $\phi(t)$, this static approximation remains valid under the assumption that the scalar field evolves slowly, and local gravitational effects dominate over cosmological time variation. Thus, we have obtained the exact solution of a spherically symmetric spacetime describing a black hole enveloped by quintessence has been obtained and is presented in the following metric form, [28-29]

$$ds^2 = -h(r)dt^2 + h(r)^{-1}dr^2 + r^2 d\theta^2 + \sin^2\theta r^2 d\phi^2, \tag{18}$$

with the radial function, $h(r)$, is formulated as

$$h(r) = 1 - \frac{2m}{r} - \frac{q}{r^{3\omega_q+1}}. \tag{19}$$

In this study, the black hole mass (m) is selected within a range that ensures the presence of an event horizon in the Schwarzschild solution modified by quintessence. In the Schwarzschild solution, the event horizon occurs when $h(r)=0$. In case $\omega_q=-1$, the quintessence includes the form of the cosmological constant and spacetime can be reduced to a Schwarzschild-de Sitter (SdS) black hole. The value of m must be large enough to ensure that the solution has an event horizon [32]. If m is too small or does not match the value of the quintessence parameter q, then the solution may become unstable or lose its physical interpretation. The choice of mass values in this study follows a numerical method to ensure that an event horizon exists, thereby enabling physically valid calculations of the Hawking temperature.

The Hawking Temperature in The Horizons of Blackhole and Cosmology

Stephen Hawking combined the general relativity with quantum field theory to explain the emission of black holes radiation, resulting in a thermal spectrum similar to blackbody radiation [33-34]. This emission is known as Hawking radiation. Hawking radiation describes the particle emission rate, portraying black holes as hot objects where the temperature is directly proportional to the surface gravity, expressed by the following equation:

$$T_0=\frac{\kappa_h}{2\pi}, \tag{20}$$

where T_0 refers to the Hawking temperature at the black hole horizon and κ_h represents the surface gravity of black hole [35]. At the black hole horizon, the surface gravity is given by

$$\kappa_h^2=\frac{1}{2}\lim_{r\to r_h}\left(D_\mu K_\nu\right)\left(D^\mu K^\nu\right). \tag{21}$$

In this expression, D_μ denotes the covariant derivative operator, r_h corresponds to the radius of the black hole horizon, and $K=\gamma_t\frac{\partial}{\partial t}$ is the timelike Killing vector field with a normalisation constant γ_t. It is important to note that although the derivative of the radial function can yield negative values for the horizon, this does not imply a physically negative temperature. The physically meaningful Hawking temperature is always non-negative because it is defined by taking the absolute value of the surface gravity:

$$\kappa_h=\frac{1}{2}\frac{1}{\sqrt{-g_{\mu\mu}g_{\nu\nu}}}\left|g_{\mu\mu,\nu}\right|_{r=r_h}. \tag{22}$$

This ensures that the temperatures T_0, T_{BH}, and T_c remain physically interpretable in all regions of spacetime affected by the quintessence. The calculation of the surface gravity of the black hole involves analysing the radial function $h(r)$ from the Schwarzschild metric with quintessence.

The Hawking temperature (T_0) based on the surface gravity of this metric can be determined by specifying the state parameter of quintessence (ω_q) which has a value range of $-1<\omega_q<-\frac{1}{3}$ [5, 21, 24, 27- 28]. This range is based on cosmological theory which shows that if $\omega_q<-\frac{1}{3}$, then dark energy plays a role as a cause of the acceleration expansion of the universe. If $\omega_q=-1$, this field becomes identical to the cosmological constant Λ. For $\omega_q > -\frac{1}{3}$, this field no longer acts as dark energy in standard cosmology. Therefore, the parameter range ω_q used in this study follows the physical constraints set by previous cosmological models. In this study, we choose $\omega_q = -\frac{2}{3}$ as a representative value of the quintessential state parameter within the range $-1 < \omega_q < -\frac{1}{3}$. This choice not only satisfies the condition for cosmic acceleration but also leads to tractable analytic solutions that exhibit horizon structure. Substituting this specific value ω_q, eq. (19) simplifies to the following radial function:

$$h(r) = 1 - \frac{2m}{r} - qr, \tag{23}$$

which produces two different horizons, namely

$$r_h = \frac{1 - \sqrt{1 - 8qm}}{2q} \tag{24a}$$

and

$$r_c = \frac{1 + \sqrt{1 - 8qm}}{2q}. \tag{24b}$$

In this context, r_h denotes the event horizon of the black hole and r_c refers to the cosmological horizon.

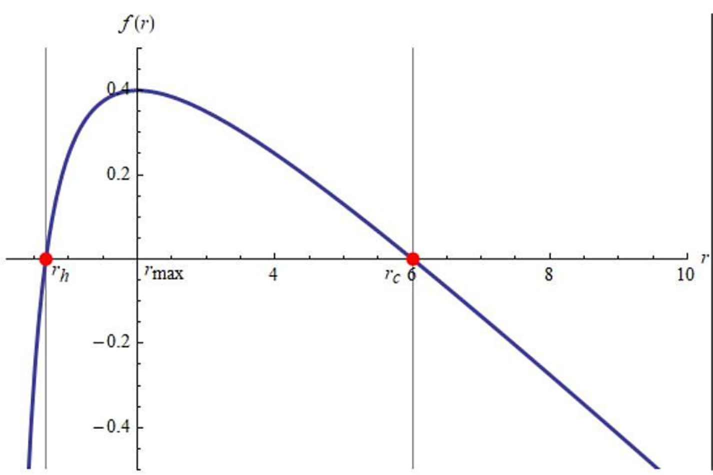

Figure 1. Graph of $f(r)$ and r with m = 0.3 and q = 0.15

In the case of a Schwarzschild black hole with quintessence, the existence of these horizons is constrained by the condition $1 - 8qm > 0$, which implies that the black hole mass must satisfy $m < 1/8q$. When the mass reaches the critical value $m = 1/8q$, the two horizons merge into a single degenerate horizon at $r_h = r_c = 1/2q$. Therefore, for a fixed value of q, the maximum allowable mass ($m_{\max}$) for which distinct horizons can exist is $1/8q$. For values of m within the range $0 < m < m_{\max}$, a static region exists between the black hole event horizon (r_h) and the cosmological horizon (r_c), as illustrated in **Figure 1**.

In addition, a shaded exclusion region is included in the parameter space plot (**Figure 2**) to explicitly indicate the boundary where no real horizons exist, corresponding to the condition $1-8qm<0$. This region highlights the unphysical domain in which the discriminant becomes negative, causing the square root in equations (24a) and (24b) to yield imaginary values. Hence, only values of q and m satisfying $1-8qm>0$ are considered valid in subsequent analyses.

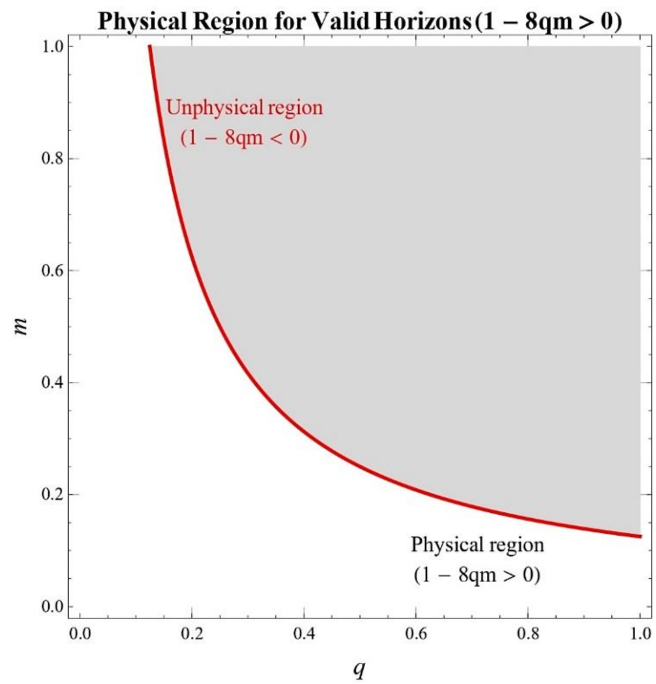

Figure 2. Physical Region for Valid Horizon ($1-8qm>0$)

It should be emphasized that the derivative may produce negative values under certain conditions for the cosmological horizon; this does not imply a physically negative temperature. The physically meaningful Hawking temperature is always non-negative because it is defined by taking the absolute value of the surface gravity. The formulation of black hole surface gravity:

$$\kappa_h = \left| \frac{1-2qr_h}{2r_h} \right|, \tag{25}$$

and the formulation of the Hawking temperature at the black hole horizon:

$$T_0 = \frac{1}{4\pi}\left|\frac{1-2qr_h}{r_h}\right|. \tag{26}$$

Black holes have very strong gravity and can attract anything, including light, so that the thermodynamics in the Schwarzschild spacetime with quintessence are not in equilibrium. In addition, this spacetime does not have an asymptotic fundamental boundary where all black hole parameters are expressed as radial functions $h(r)$ in the metric [8]. This function interpolates the event horizon of the black hole (r_h) and the cosmological horizon (r_c), which have maximum values at the midpoint as shown in the following equation:

$$r_0^2 = \frac{2m}{q}, \tag{27}$$

where r_0 represents the maximum point of the radial function $h(r)$, so that the radial function becomes

$$h(r_0) = 1 - 2qr_0. \tag{28}$$

When the value of the quintessence parameter (q) is small, the quintessence contribution to the radial function $h(r)$ becomes weak and approaches the Schwarzschild solution. The two horizons (r_h and r_c) will remain in positions close to the Schwarzschild configuration without quintessence, that is, they are only affected by the mass (m). For small q, it only gives a shift in the radius, which may be difficult to detect significantly. In addition, the positions of the horizons are less significantly altered by quintessence dark energy than by the cosmological constant (Λ) as shown in [8]. As a result, the two horizons remain relatively far apart from each other so that the thermodynamics for each horizon can be analyzed independently [9]. However, with increasing black hole mass (m), the two horizons eventually draw closer and coincide at a critical boundary ($r_h = r_c$). An observer interacting with both horizons at any point in the causal region ($r_h < r < r_c$) will never be in thermodynamic equilibrium.

To avoid singularities, it is necessary to normalize to ensure that the temperature value remains valid throughout spacetime. Because of this condition, the Hawking temperature formulation at the black hole horizon becomes

$$T_{BH} = \frac{1}{4\pi}\left|\frac{1-2qr_h}{\sqrt{h(r_0)}r_h}\right|. \tag{29}$$

From this equation it is shown that at a distance r_0, the influence of the horizons of black hole and cosmological cancel each other out and the point is closest to the asymptotically flat boundary. Then, the surface gravity of the cosmological horizon (κ_c) in the same way has a temperature according to the following equation:

$$T_c = \left|-\frac{\kappa_c}{2\pi}\right| = \frac{1}{4\pi}\left|-\frac{1-2qr_c}{r_c}\right|. \tag{30}$$

Results and Discussion

The existence of dark energy (quintessence) is only indirectly detected through the global gravitational effect in the universe. This means that the influence of dark energy will remain if the universe expands because the gravity in the universe will cause a negative force (repulsive force) that will accelerate the universe expansion [38]. The existence of dark energy is supported by several cosmological observations, such as observations of Type Ia Supernovae, Cosmic Microwave Background (CMB) observations, Large-Scale Structure (LSS), Baryonic Acoustic Oscillations (BAO), and gravitational lens effects [4-5, 34-36]. From the results of the Hawking temperature calculations obtained previously, the presence of dark energy (quintessence) can change the geometry of space-time, increase the scale of the cosmological horizon, and reduce the surface gravity of black holes, which directly lowers the Hawking temperature. This effect has implications for the dynamics of black holes, where Hawking radiation becomes weaker in space-time with quintessence and can extend the evaporation time of black holes.

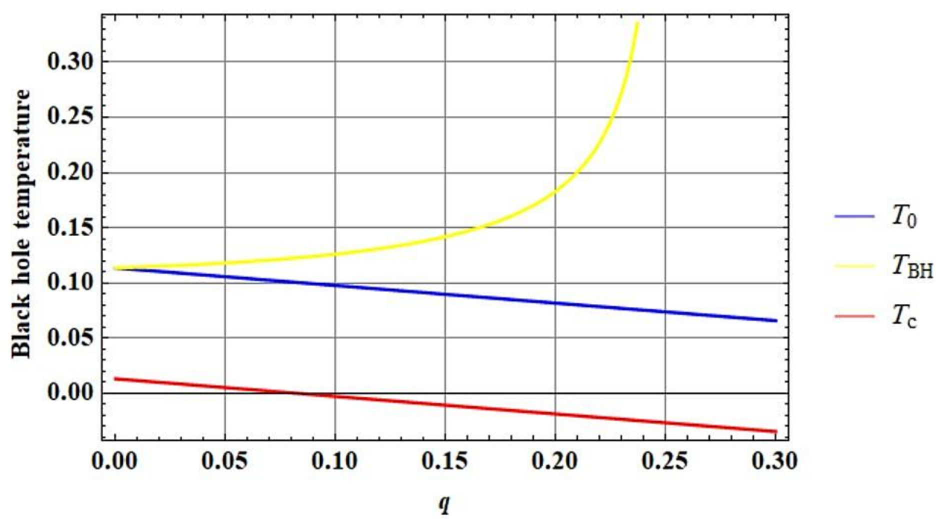

Figure 3. Hawking temperature spectrum at the black hole horizon radius $r_h \approx 0.7$: without normalization (T_0), with normalization (T_{BH}) and Hawking temperature spectrum at the black hole cosmological radius $r_c \approx 6$ (T_c) with $m = 0.3$

Figure 3 shows the relationship between the quintessence normalization parameter (q) on the horizontal axis and three types of black hole temperatures on the vertical axis. The standard Hawking temperature without normalization (T_0) is shown as the blue line, at the black hole horizon r_h (T_{BH}) is shown as the yellow line, at the cosmological horizon r_c (T_c) is shown as the red line. The T_0 line decreases linearly with q. This is in accordance with the expectation that increasing the quintessence contribution (larger q) will weaken the local gravitational effect, so that the Hawking temperature decreases. T_0 can even be negative if not constrained. The temperature at the cosmological horizon (T_c) also shows a linear decrease with q and approaches negative at $q > 0.07$ which is physically meaningless and indicates that the cosmological horizon cannot be defined properly for high q. This is consistent with the prediction that the quintessence field dominates on large scales and expands the cosmological horizon, thus decreasing its gravitational surface and temperature. Unlike the previous two temperatures, the temperature at the black hole horizon (T_{BH}) shows a significant non-linear increase. This increase occurs because the black hole horizon shrinks with increasing q, which causes its gravitational surface acceleration (κ) to increase and results in higher thermal radiation. This effect reflects the transition from local gravitational dominance to quintessence field dominance, which shrinks the horizon and increases local radiation. The exponential increase of T_{BH} at a certain value of q also indicates that there is an upper limit on the parameter q for the system to remain physical. This shows that in this range, the effect of quintessence on deep horizons is very strong and can increase the temperature.

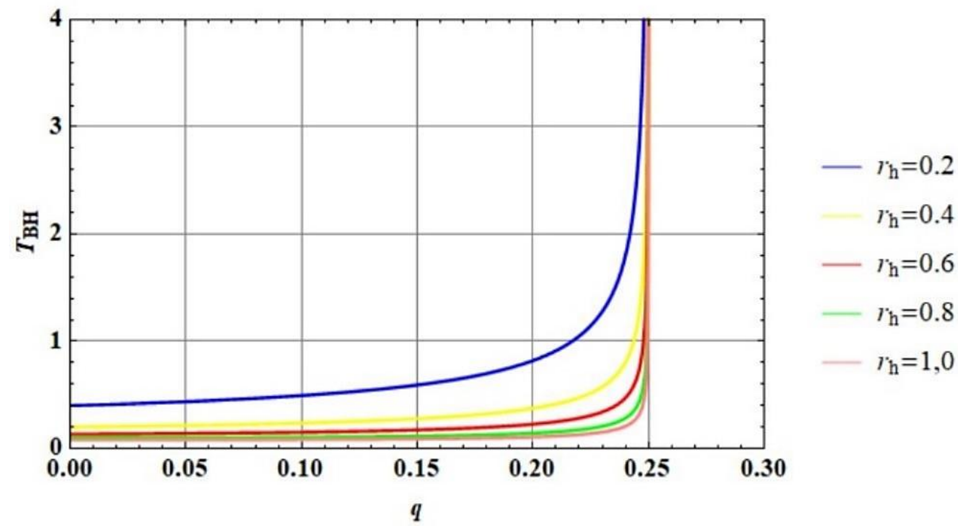

Figure 4. Hawking temperature spectrum for variations in the black hole horizon radius r_h with normalization (T_{BH}) with m = 0.3

The relationship between the Hawking temperature at the normalization black hole horizon (T_{BH}) with the quintessence parameter (q) is shown in **Figure 4**. For small values of q (close to 0), T_{BH} increases very slowly at all values of r_h. The quintessence contribution is still weak, and the system still closely resembles a regular Schwarzschild black hole. For larger values of r_h, T_{BH} temperature of the black hole remains relatively low for the same q. As q approaches the upper limit in this plot (about 0.25), T_{BH} increases sharply. The rate of increase in temperature with q becomes more pronounced as the event horizon radius r_h decreases, with $r_h \approx 0.2$ showing the sharpest increase. This means that small black holes are more sensitive to the effects of dark energy. This plot illustrates a theoretical model in which the quintessence temperature of a black hole is calculated as a function of q and its horizon radius, which allows us to explore the properties of black holes in a modified theory of general relativity.

From **Figure 5**, we observe that the Hawking temperature at the cosmological horizon (T_c) decreases by increasing the quintessence parameter (q), indicating an inverse relationship between T_c and q. Moreover, higher values of r_c lead to lower values of T_c for a given q. As q increases from 0 to 1, T_c decreases linearly for a fixed r_c. Higher values of r_c lead to lower T_c for the same q. Physically, the decrease in Tc with increasing q reflects that the presence of quintessence field associated with dark energy forms contributes to "cooling" the cosmological horizon. The increase in q indicates a greater dominance of dark energy, which results in a stronger acceleration of the universe's expansion and affects the structure of the horizon. In this context, a larger cosmological horizon r_c indicates a wider system, so its T_c becomes lower. This phenomenon is analogous to the effect of the expansion of the universe which causes the cosmological background temperature to become smaller. Overall, the decrease in T_c with respect to q and r_c illustrates the cooling effect of the universe due to the contribution of dark energy in the framework of space-time thermodynamics.

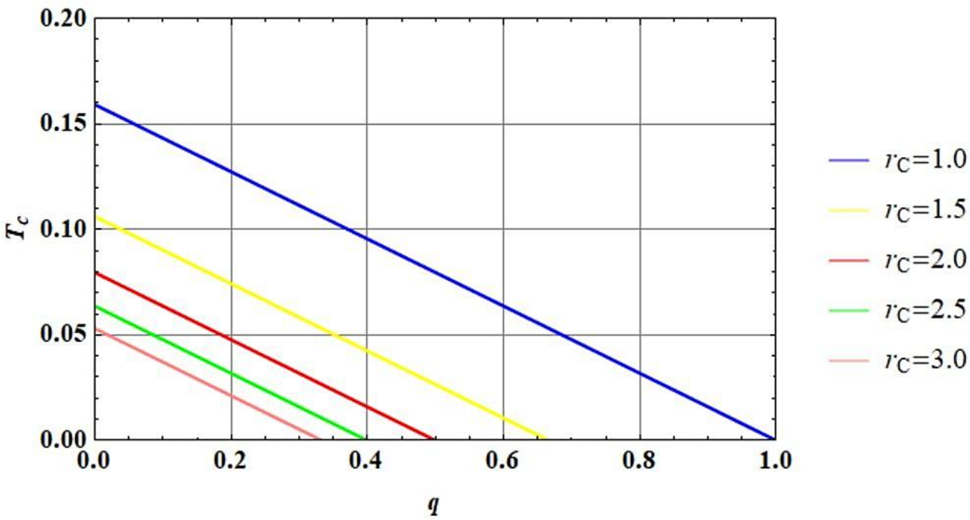

Figure 5. Hawking temperature spectrum for variations in the cosmological horizon radius r_c (T_c) with m = 0.3

Furthermore, the Hawking temperature at the black hole horizon with normalization (T_{BH}) and at the cosmological horizon (T_c) spectrum at mass (m) variation can be seen in **Figure 5**. It shows that T_{BH} decreases significantly as m increases. This observation is consistent with Hawking's original formulation, in which the black hole temperature is inversely proportional to its mass, $T_{\mathrm{BH}} \propto 1/m$ for a Schwarzschild black hole. Physically, this implies that smaller black holes are hotter, while larger black holes are cooler, a key thermodynamic feature of black hole evaporation.

In our model, this classical behavior is preserved even in the presence of quintessence, as shown in the numerical results. However, it is important to note that the presence of quintessence modifies the geometry of spacetime through an additional term in the metric function. This modification affects the surface gravity (κ), which under standard conditions is proportional to $1/m$. With quintessence, this relation may be altered depending on the state parameter (ω_q) and the normalization factor (q), potentially leading to deviations from the Schwarzschild case. While our current study numerically captures how T_{BH} varies with m under the influence of quintessence, a detailed analytical derivation of the modified surface gravity κ and its impact on temperature is not addressed in the present work and will be explored in subsequent studies.

Additionally, the behavior of T_c is also depicted in **Figure 6**. Unlike T_{BH}, T_c remains nearly constant and very close to zero across the examined mass range. This suggests that the cosmological horizon temperature is largely insensitive to changes in the black hole mass and consistently much lower than T_{BH}, further emphasizing the thermal asymmetry between local (black hole) and large-scale (cosmological) horizons in the influence of quintessence.

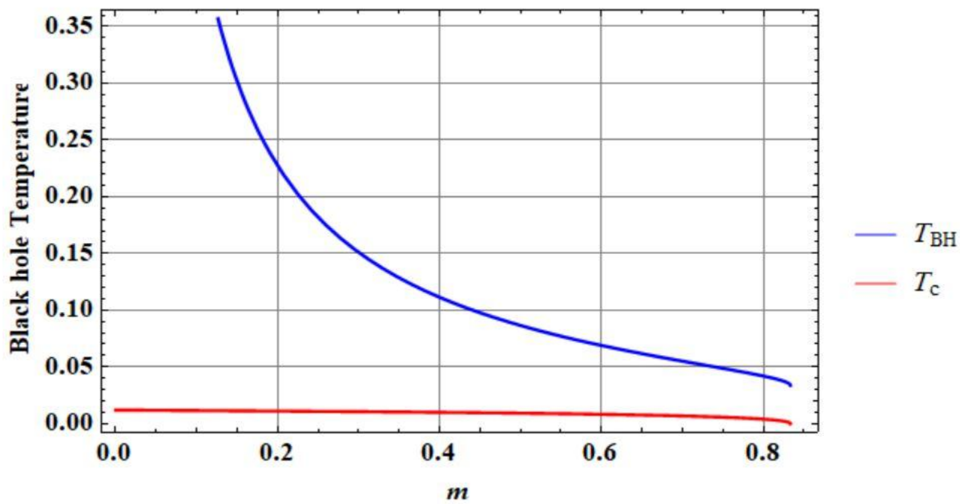

Figure 6. Graph of the Hawking temperature at the black hole horizon with normalization (T_{BH}) and at the cosmological horizon (T_c) at mass (m) variation and q = 0.15

Conclusion

Represented by a quintessence, dark energy plays an important role in influencing Hawking radiation and the Schwarzschild black holes temperature. From the linear decrease in the standard Hawking temperature without normalization (T_0) and the nonlinear decrease in the normalized black hole temperature at the black hole event horizon (T_{BH}) indicate a direct relationship between dark energy and black hole dynamics. In this study, we investigated the variation of the normalization of quintessence parameter q to examine the physical behaviour of the black hole solutions within the allowed parameter regime and our analysis does not aim to redefine or reinterpret normalization in the context of quantum theory. When q is small, quintessence only provides minor corrections to the Schwarzschild horizon and thermodynamics of the black hole. With increasing q, the Hawking temperature at the black hole horizon (r_h) decreases, while simultaneously causing an increase in the temperature at the cosmological horizon (r_c), reflecting the growing influence of dark energy on large-scale spacetime. Physically, the distribution of dark energy (quintessence) affects both horizons by weakening the local gravitational effects near the black hole while enhancing the cosmological effects at larger scales. Thus, quintessence affects not only individual black holes but also the entire structure of the universe. Moreover, the temperature quantities T_0, T_{BH}, and T_c are physically relevant because they are derived from the surface gravity at r_h and r_c, both of which are modified by the quintessence. This analysis provides thermodynamic insight into the structure of black holes' spacetime under the influence of dark energy. It supports the interpretation that Hawking radiation and horizon thermodynamics may offer a novel avenue for probing the nature of dark energy through gravitational phenomena. We chose $\omega = -2/3$ in this study because it simplifies the analysis through a linear radial function, represents a transitional regime between $-1/3 < \omega < -1$, and is consistent with prior studies on black holes with quintessence. In future work, we will explore a broader range of ω that would enhance the generality of the results (including phantom $\omega < -1$).

Acknowledgment

This research was funded through a grant provided by the Faculty of Science, Institut Teknologi Sumatra (Itera).

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