Indonesian Physical Review

Volume 7 Issue 1, January 2024

P-ISSN: 2615-1278, E-ISSN: 2614-7904

Rotation component of robotic manipulator motion in 4-DoF by the quaternion method

Luh Putu Budi Yasmini[1], Wardah Nur Isna[1], Nurfa Risha[1]

[1] *Faculty of Mathematics and Science, Universitas Pendidikan Ganesha, Indonesia.*

Corresponding Authors E-mail: budi.yasmini@undiksha.ac.id

Article Info

Article info:
Received: 20-09-2023
Revised: 14-11-2023
Accepted: 23-01-2024

Keywords:
Robotic manipulator; degree of freedom; quaternion;

How To Cite:
L. P. B. Yasmini, W. N. Isna, and N. Risha, "Rotation component of robotic manipulator motion in 4-DoF by the quaternion method", Indonesian Physical Review, vol. 57 no. 1, p 119-124, 2024.

DOI:
https://doi.org/10.29303/ipr.v7i1.276.

Abstract

Most of the study of robots is about the system. Because of that, we analyze the motion of the robot by using rigid body motion concepts. This study aims to analyze the rotation components of a robotic manipulator with four degrees of freedom (4-DoF) by using the quaternion method of manual analysis so that the end effector of the robotic manipulator under study is obtained. The research is theoretical and was conducted through a review and analysis of related literature. Based on the literature review, the robotic manipulator is a robot with a shape resembling an arm. The robotic manipulator consists of bases, links, joins, and an end effector. In terms of physics, the kinematics of robot motion can be studied based on the concept of rigid body motion. The research shows that the quaternion method can be used to determine the end effector of the rotation component of the robotic manipulator, and the steps are shorter than analyses by using screw and twist theory. Based on this case, the end effector of the rotation component depends on the initial state and the total rotation angle of each join.

Introduction

Today, the development of technology is very fast, and everything seems to run so fast. Particularly related to the development and utilization of robot technology, both on a small and industrial scale [1], [2]. One type of robot that is widely used on an industrial scale is the robotic manipulator. The development of robot manipulators is expected to make work easier and more efficient [3], [4]. This type of robot consists of rigid arms connected in series. The arms are connected through joints that can move in rotation and translation related to the ability of the robot to move freely, which is called degree of freedom (DOF) [5]. Many studies have been carried out related to the development of robotic manipulator systems [2]–[4], [6]–[11]. However, most of the studies carried out are related to the system design and sensors used. Based on that, this research is focused on analyzing robot motion in terms of related physics concepts.

Generally, robot motion can be analyzed based on the concept of motion in physics, i.e., the concept of rigid body motion. The motion of the system is said to be rigid body motion if the distance between points on each body does not change during the motion. This concept is becoming the basic concept for analyzing the motion of robot manipulators [5], [12]. This research is a theoretical study conducted through a review of related literature. This study aims to analyze the rotational components of a robotic manipulator with four degrees of freedom (4-DoF) to obtain the end effector of the robot. In this study, the method used to analyze the rotation components of the robot manipulator motion is the quaternion method. This method is one way to analyze the only rotational component of robot motion. The quaternion method is a method that simplifies analysis when it involves rotations in Euclidean space ($\mathbb{R}^3$), where quaternion is a vector consisting of one scalar element and three vector elements that is used not only to analyze the robot motion but also molecular modeling andthe other dynamical simulation [9], [13]–[16].

Quaternion Method

A vector in $\mathbb{R}^3$ can be expressed as a quaternion in the rotation of a rigid body. Quaternions can be used to represent the rotational component of the motion of a rigid body.
Determined that a quaternion is a vector quantity which can be written as follows [5], [13]:

$$Q = q_0 + q_1\hat{\imath} + q_2\hat{\jmath} + q_3\hat{k} \quad (1)$$

where q_0 is the scalar component of Q and $\boldsymbol{q} = (q_1, q_2, q_2)$ are the vector components.
It is assumed that the rotation of a rigid body is in a plane perpendicular to a rotating axis, so it can be stated that.

$$cos^2\frac{\alpha}{2} = {q_0}^2$$
$$sin^2\frac{\alpha}{2} = \|{\boldsymbol{q}_n}^2\|$$

where n = 1,2,3 and $\alpha = [0, \pi]$. Moreover, a quaternion that can describe the rotational component of the motion of a rigid body can be written as:

$$q = \cos\frac{\alpha}{2} + \widehat{\omega}\sin\frac{\alpha}{2} \quad (2)$$

where $\widehat{\omega}$ is an operator written $\omega\,\times$, and $\omega \in \mathbb{R}^3$ be a unit vector that describes the direction of rotation.
If there are two vectors $\boldsymbol{v}, \boldsymbol{w} \in \mathbb{R}^3$ that is related to a transformation matrix, then fulfilled.

$$\begin{aligned}\boldsymbol{w} = q\mathbf{v}q^* &= (q_0 + \boldsymbol{q}_n)(0 + \mathbf{v})(q_0 + \boldsymbol{q}_n) \\ &= (2{q_0}^2 - 1)\mathbf{v} + 2(\boldsymbol{q}_n \cdot \mathbf{v})\boldsymbol{q}_n + 2q_0(\boldsymbol{q}_n \times \mathbf{v}) \quad (3)\end{aligned}$$

or in matrix form it can be written as.

$$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} 2{q_0}^2 - 1 + 2q_1^2 & 2q_1q_2 - 2q_0q_3 & 2q_1q_3 + 2q_0q_2 \\ 2q_1q_2 + 2q_0q_3 & 2{q_0}^2 - 1 + 2q_2^2 & 2q_2q_3 - 2q_0q_1 \\ 2q_1q_3 - 2q_0q_2 & 2q_2q_3 + 2q_0q_1 & 2{q_0}^2 - 1 + 2q_3^2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_2 \end{bmatrix} \quad (4)$$

Kinematics of Robotic Manipulator with 4-DoF

The modeling of robotic manipulator with 4-DoF studied is illustrated as in Figure 1.

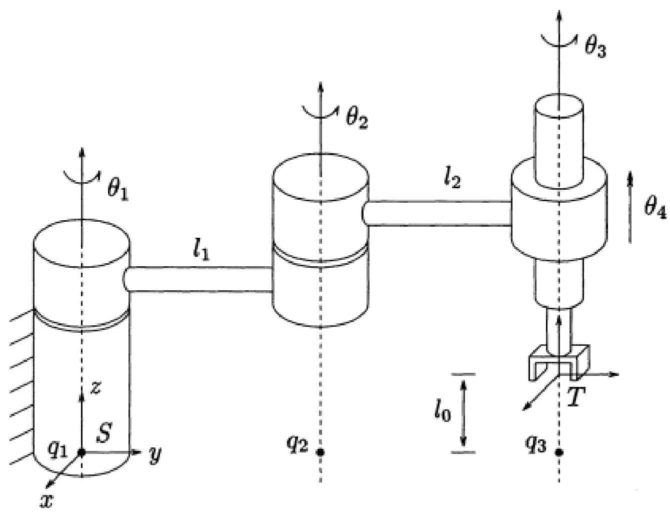

Figure 1. Modeling of Robotic Manipulator with 4-DoF [17]

The characteristics of robotic manipulator being analyzed is the robot has 4 degrees of freedom. As in Figure1., the robotic manipulator has 3 rotational motion capabilities i.e., $\theta_1, \theta_2, \theta_3$ (which are limited to rotation only about the z-axis) in each joint so that it is called a revolute joint. Also, it has 1 translational movement i.e., θ_4 in the last joint is called a prismatic joint (θ_4 has length units).

Rotation component of robotic manipulator motion in 4-DoF by quaternion method

Based on Figure 1. and (4), the quaternion that describes the rotation motion of the robot can be written as follows,

$$q = q_0 + \boldsymbol{q}_n = q_0 + q_i\hat{\imath} + q_2\hat{\jmath} + q_3\hat{k}$$
$$q = \cos\frac{\alpha}{2} + \hat{\omega}\sin\frac{\alpha}{2} = \cos\frac{\alpha}{2} + (0,0,1)\sin\frac{\alpha}{2}$$
$$q = \cos\frac{\alpha}{2} + 0\hat{\imath} + 0\hat{\jmath} + \sin\frac{\alpha}{2}\hat{k} \quad (5)$$

where $(\theta_1 + \theta_2 + \theta_3) = \alpha$ and $\hat{\omega} = (0,0,1)$ (which describes the rotation about z-axis). So, to determine the end effector of the rotation component of that robotic manipulator are,

$$\begin{bmatrix} w_x \\ w_y \\ w_z \end{bmatrix} = \begin{bmatrix} 2{q_0}^2 - 1 + 2q_1^2 & 2q_1q_2 - 2q_0q_3 & 2q_1q_3 + 2q_0q_2 \\ 2q_1q_2 + 2q_0q_3 & 2{q_0}^2 - 1 + 2q_2^2 & 2q_2q_3 - 2q_0q_1 \\ 2q_1q_3 - 2q_0q_2 & 2q_2q_3 + 2q_0q_1 & 2{q_0}^2 - 1 + 2q_3^2 \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix}$$

$$= \begin{bmatrix} 2\,.\cos^2\frac{\alpha}{2} - 1 + 2.0^2 & 2.0.0 - 2.\cos\frac{\alpha}{2}.\sin\frac{\alpha}{2} & 2.0.\sin\frac{\alpha}{2} + 2.\cos\frac{\alpha}{2}.0 \\ 2.0.0 + 2.\cos\frac{\alpha}{2}.\sin\frac{\alpha}{2} & 2.\cos^2\frac{\alpha}{2} - 1 + 2.0^2 & 2.0.\sin\frac{\alpha}{2} - 2.\cos\frac{\alpha}{2}.0 \\ 2.0.\sin\frac{\alpha}{2} - 2.\cos\frac{\alpha}{2}.0 & 2.0.\sin\frac{\alpha}{2} + 2.\cos\frac{\alpha}{2}.0 & 2.\cos^2\frac{\alpha}{2} - 1 + 2.\sin^2\frac{\alpha}{2} \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix}$$
$$= \begin{bmatrix} 2\,.\cos^2\frac{\alpha}{2} - 1 & -2.\cos\frac{\alpha}{2}.\sin\frac{\alpha}{2} & 0 \\ 2.\cos\frac{\alpha}{2}.\sin\frac{\alpha}{2} & 2.\cos^2\frac{\alpha}{2} & 0 \\ 0 & 0 & 2\left(\cos^2\frac{\alpha}{2} + \sin^2\frac{\alpha}{2}\right) - 1 \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} \quad (6)$$

Case application

Based on Figure 1., if it is determined that the length of the robot arm for each joint is as follows: l_1 and l_2 = 10 cm, l_0 = 5cm, dan $\theta_1, \theta_2, \theta_3$ = 30° (which are rotated about the z-axis), and also θ_4 = 5 cm. Then the following is the end effector value of the rotation component of the robot manipulator is

$$\begin{bmatrix} w_x \\ w_y \\ w_z \end{bmatrix} = \begin{bmatrix} \left(2\,.\cos^2\frac{90}{2} - 1\right).1 + \left(-2.\cos\frac{90}{2}.\sin\frac{90}{2}\right).0 \\ \left(2.\cos\frac{90}{2}.\sin\frac{90}{2}\right).1 + \left(2.\cos^2\frac{90}{2}\right).0 \\ \left(2\left(\cos^2\frac{90}{2} + \sin^2\frac{90}{2}\right) - 1\right).0 \end{bmatrix} = \begin{bmatrix} (2(0{,}5) - 1).1 + (-2.(0{,}707).(0{,}707)).0 \\ (2.(0{,}707).(0{,}707)).1 + (2.(0{,}5)).0 \\ (2((0{,}5) + (0{,}5)) - 1).0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad (7)$$

where $\alpha = \theta_1 + \theta_2 + \theta_3 = 30° + 30° + 30° = 90°$ and chosen that $(v_x, v_y, v_z) = (1{,}0{,}0)$, is shows that the initial position of the robot is in (1,0,0), respectively in 3D Cartesian coordinate, after the rotation $\theta_1, \theta_2, \theta_3$, based on (7), the rotation component of end effector in this case is (0,1,0). Metode quaternion that used in this analyze is to find the rotation component of end effector, in this case, depend on the $\theta_1, \theta_2, \theta_3$, and the initial position (v_x, v_y, v_z), but the translation component is not analyze. Next, we analyze some cases for variation (v_x, v_y, v_z) and $\theta_1, \theta_2, \theta_3$ based on Tabel 1 and Tabel 2.

Table 1. End effector of 4-DoF Manipulator in variation of initial position

No	Variation of (v_x, v_y, v_z)	Rotational component of end effector
1	1,0,0	0,1,0
2	1,1,0	-1,2,0
3	1,0,1	0,1,1
4	1,0,2	0,1,2
5	1,2,0	-2,3,0

Table 2. End effector of 4-DoF Manipulator in variation of rotational angel (in $\alpha = 90^{\circ}$)

No	**Rotational angel ($\theta_1, \theta_2, \theta_3$)**	**Rotational component of end effector**
1	$10^0, 20^0, 60^0$	0,1,0
2	$15^0, 30^0, 45^0$	0,1,0
3	$20^0, 40^0, 30^0$	0,1,0
4	$25^0, 50^0, 15^0$	0,1,0
5	$30^0, 60^0, 0^0$	0,1,0

Based on Tabel 2, we found that for the case with the same amount of rotational angel for each join ($\alpha = \theta_1 + \theta_2 + \theta_3$), the rotational component of the end effector is same.

Conclusion

To conclude, it is found that the quaternion method can be used to determine the end effector of the rotational component of a robotic manipulator. Based on the case studied, the end effector of the rotation component depends on the initial state and the total rotation angle of each joint.

Acknowledgment

We want to say thank you to Universitas Pendidikan Ganesha for supporting this research by Penelitian Dasar grant with a contract number: 687/UN48.16/LT/2023.

References

[1] M. Irsan Kasau and S. Aminah Dinayati Ghani, "ANALISIS KINEMATIK ROBOT KOORDINAT BOLA, ENAM DERAJAT KEBEBASAN DENGAN METODE QUATERNION & ROTATION VECTOR," 2018.

[2] V. G. Pratheep, M. Chinnathambi, E. B. Priyanka, P. Ponmurugan, and P. Thiagarajan, "Design and Analysis of six DOF Robotic Manipulator," *IOP Conf. Ser. Mater. Sci. Eng.*, vol. 1055, no. 1, p. 012005, Feb. 2021, doi: 10.1088/1757-899x/1055/1/012005.

[3] D. Suddin and B. Nasrullah, "Rancang Bangun Robot Manipulator Yang Bergerak Secara Translasi Dan Rotasi," vol. 1, pp. 151–156, 2017, [Online]. Available: http://snp2m.poliupg.ac.id/2017/

[4] M. Faiq, W. Satriatama, and L. Halim, "Perancangan Awal dan Simulasi Lengan Robot 3 Derajat Kebebasan Berbasis Arduino yang Dikontrol dengan Aplikasi," *J. Mech. Eng. Mechatronics*, vol. 7, no. 2, pp. 118–130, 2022.

[5] R. M. Murray, Z. Li, and S. S. Sastry, *A Mathematical Introduction to Robotic Manipulation*, vol. 84, no. 788. 1994.

[6] J. Andika and K. S. Salamah, "Analisis kinematik pada robot hexapod," *J. Teknol. Elektro*, vol. 9, no. 2, pp. 83–91, 2018.

[7] J. Gallardo-Alvarado, R. Rodriguez-Castro, L. Perez-Gonzalez, and C. R. Aguilar-Najera, "Kinematics of the 3(RPSP)-S fully spherical parallel manipulator by means of screw theory," *Robotics*, vol. 7, no. 2, 2018, doi: 10.3390/robotics7020029.

[8] D. Ge, "Kinematics modeling of redundant manipulator based on screw theory and Newton-Raphson method," *J. Phys. Conf. Ser.*, vol. 2246, no. 1, p. 012068, Apr. 2022, doi:

10.1088/1742-6596/2246/1/012068.
[9] S. Kucuk and Z. Bingul, "Robot Kinematics: Forward and Inverse Kinematics," in *Industrial Robotics: Theory, Modelling and Control*, Pro Literatur Verlag, Germany / ARS, Austria, 2006. doi: 10.5772/5015.
[10] A. E. Salman and M. Roman, "Robot Kinematics," 2022. [Online]. Available: https://www.researchgate.net/publication/365193182
[11] M. Gouasmi, M. Ouali, B. Fernini, and M. Meghatria, "Kinematic modelling and simulation of a 2-R robot using solidworks and verification by matlab/simulink," *Int. J. Adv. Robot. Syst.*, vol. 9, Dec. 2012, doi: 10.5772/50203.
[12] L. P. B. Yasmini and I. G. A. Gunadi, "KAJIAN AWAL PENERAPAN KONSEP MEKANIKA PADA GERAK ROBOT DENGAN ANALISIS ALJABAR," 2012.
[13] J. B. Kuipers, "Quaternions and rotation sequences _ a primer with applications to orbits, aerospace, and virtual reality," 1999.
[14] V. Artale, C. L. R. Milazzo, and A. Ricciardello, "An example of quaternion parameterization for dynamical simulations," *J. Phys. Conf. Ser.*, vol. 490, no. 1, pp. 6–10, 2014, doi: 10.1088/1742-6596/490/1/012005.
[15] R. Anderson and G. C. Joshi, "Quaternions and the Heuristic Role of Mathematical Structures in Physics," *Phys. Essays*, vol. 6, no. 2, pp. 308–319, 1993, doi: 10.4006/1.3029063.
[16] C. F. F. Karney, "Quaternions in molecular modeling," *J. Mol. Graph. Model.*, vol. 25, no. 5, pp. 595–604, 2007, doi: 10.1016/j.jmgm.2006.04.002.
[17] R. M. Murray, Z. Li, and S. S. Sastry, *A Mathematical Introduction to Robotic Manipulation Richard*, vol. 84, no. 788. 1994.