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Determination of rupture directivity using the Levenberg-Marquardt Algorithm: A case study Philippine earthquake (15 December 2019 M_w 6.8)

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Abstract

Earthquake source mechanisms are used to determine local geological characteristics and hazard mitigation. There are several approaches to determine the mechanism of earthquake. In this paper, the relative time of the rupture duration is applied to determine rupture directivity. In determining rupture directivity, the Levenberg-Marquardt (LM) method is proposed to solve the inversion problem. To test the reliability of this method, teleseismic data with an epicentral distance of 40° from the Philippine earthquake on 15 December 2019 M_w 6.8, which had a good seismic station distribution with a total of 35 stations, is used. Telesismic data from each station is filtered in the range of 0.25 to 1 Hz to obtain an accurate rupture duration. Furthermore, the rupture duration data set was inverted using the LM method to obtain the direction of earthquake rupture. The results obtained by the curve fitting using the LM method had a good agreement between the observed data and the calculated data. From the curve fitting results, the rupture propagated in the NW direction with azimuth 320.60° ± 2.30°, and this had the same results from previous studies. Therefore, from rupture directivity, the actual fault plane of this earthquake was NP1 which had a strike/dip/rake value of 321°/75°/13°, respectively. The results indicate that the Philippines earthquake of 15 December 2019 had the SE-NW fault orientation, which is part of the Cotabato fault system. The implication of this research is for a preliminary study related to the characteristics of earthquake rupture in areas that have a high level of seismicity. Thus, local residents can avoid areas where ruptures propagate when carrying out earthquake mitigation.

Introduction

Four earthquakes occurred consecutively in Mindanao, Philippines, from October to December 2019, which had M_w more than 6.4 (see Figure 1). The Philippine Institute of Volcanology and Seismology (PHIVOLCS) [1] report that, the first earthquake (yellow star) occurred on October 16, 2019 with M_w 6.4 was located at a depth of 14.1 km. Furthermore, on October 29, 2019 the second earthquake (blue star) in the northeast of the first earthquake had an M_w of 6.6 with a depth of 14.9 km. The third earthquake (green star) occurred on October 31, 2019 with a hypocenter at a depth of 10 km. The last earthquake is a fourth earthquake (red star) that occurred on 15 December 2019. While The United States Geological Survey (USGS) reported that, this earthquake had a magnitude of 6.8 with a hypocenter of 18 km [2]. In addition, the fourth earthquake has the same rupture direction as in the first earthquake. The USGS also reports that it is not only the aftershocks of the first earthquake but that the two earthquakes are connected. The relationship between the four earthquakes is still unknown, especially in the fourth earthquake on 15 December 2019, which had the greatest magnitude of the other three earthquakes. It is essential to know the source of the earthquake's mechanism to provide further knowledge about local geological conditions and carry out disaster mitigation. It is considering that the last earthquake occurred at the junction area between the Sunda Plate and the Philippine Sea Plate. Some parts of the convergence movement between these plates are caused by the Philippines fault, which slip movement of about (33 ± 11) mm/year [3].

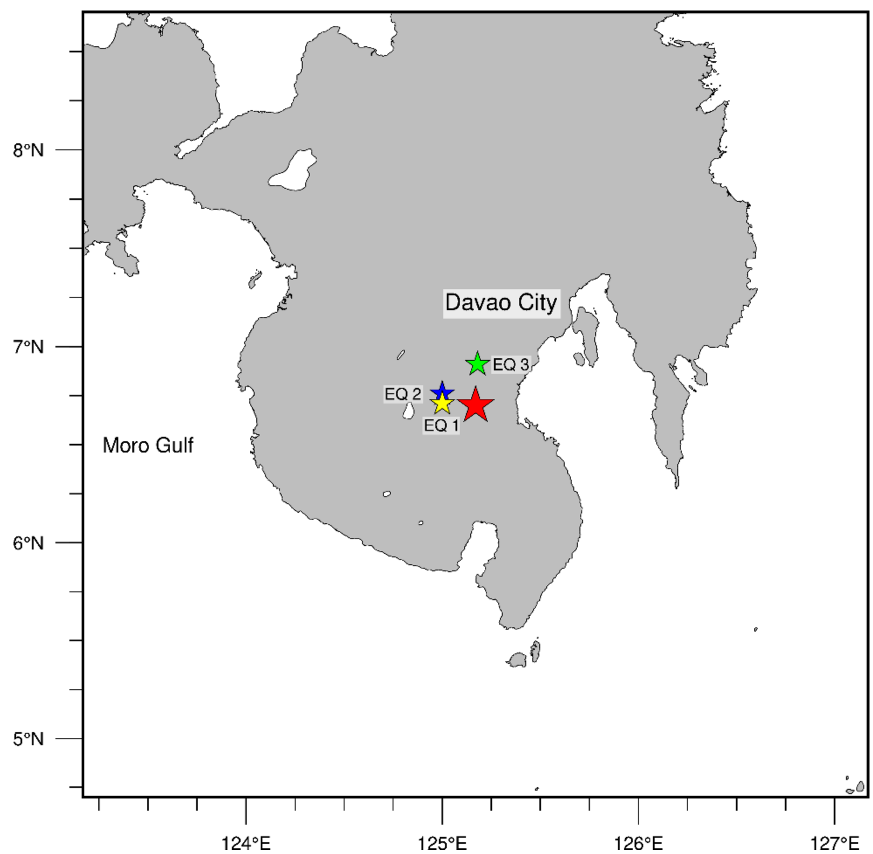

Figure 1. Four earthquakes in Mindanao Philippines

Analysis of the rupture directivity can provide critical knowledge of the mechanism of earthquake sources, one of which is to determine the actual fault plane [4-6]. Rupture directivity is a parameter that provides information regarding the rupture duration, the amplitude of the rupture, and the direction of rupture propagation, carrying the amount of energy from the earthquake source that can be identified by the azimuthal pattern recorded on the station network [7]. Wang et al. [6] analyzed the direction of the earthquake rupture that occurred on November 25, 2018 in the Taiwan Strait using the relative location differences between the hypocentral and centroid of the main earthquake and selected several aftershocks as reference events. The result is that rupture directivity can determine the actual fault plane of the earthquake that occurred. Madlazim et al. [5] use the rupture duration to determine the rupture directivity of the Palu earthquake on September 28, 2018. The result is that the rupture duration recorded on a seismic station in the direction of rupture propagation will produce a short duration, while seismic stations in the opposite direction of propagation rupture will result in a long rupture duration. Lee et al. [4] used snapshot analysis (movement of the rupture) to determine the propagation of the rupture. This analysis shows that the rupture propagation can be seen from the abnormal ground movement, which indicates a robust directional effect.

In this study determines rupture directivity of the 15 December 2019 earthquake and also determines the actual fault plane. The rupture directivity can be determined using the Levenberg-Marquardt (LM) inversion method by the curve fitting between the calculated and observed rupture duration. The advantage of the LM method is that it can find parameter solutions faster than using other non-linear inversion methods such as the Gauss-Newton method and the Steepest-Descent method. Furthermore, the results were validated with USGS data, Global Centroid Moment Tensor (GCMT), and Interferometric Synthetic Aperture Radar (InSAR).

Experimental Method

Estimation of Rupture Duration

The rupture duration was obtained from the P-wave data from each station. The data used is teleseismic wave data with an epicentral distance of 40^o consisting of 35 stations. The azimuth distribution of the seismic stations used in this study is evenly distributed around the earthquake (see Figure 2), thus the direction of rupture propagation can be estimated accurately. The rupture directivity can be determined through the relationship between rupture duration and the azimuth angle of seismic station [8]. The rupture duration can be estimated using the following equation:

$$T_{dur} = \frac{L}{V_R} - \frac{L}{V} cos(\varphi - \lambda^*) \quad (1)$$

Where T_{dur} indicates the duration of the rupture, L denotes the length of the rupture, V_R is the rupture speed, V is the seismic wave velocity, φ represent the azimuth angle of the seismic station, while λ^*indicates the azimuth angle of the direction of propagation of the rupture. Equation (1) can be interpreted that if the difference between φ and λ^* is equal to zero, it will produce a minimum rupture duration value, this can be used as a determinant of the direction of the rupture of an earthquake [8].

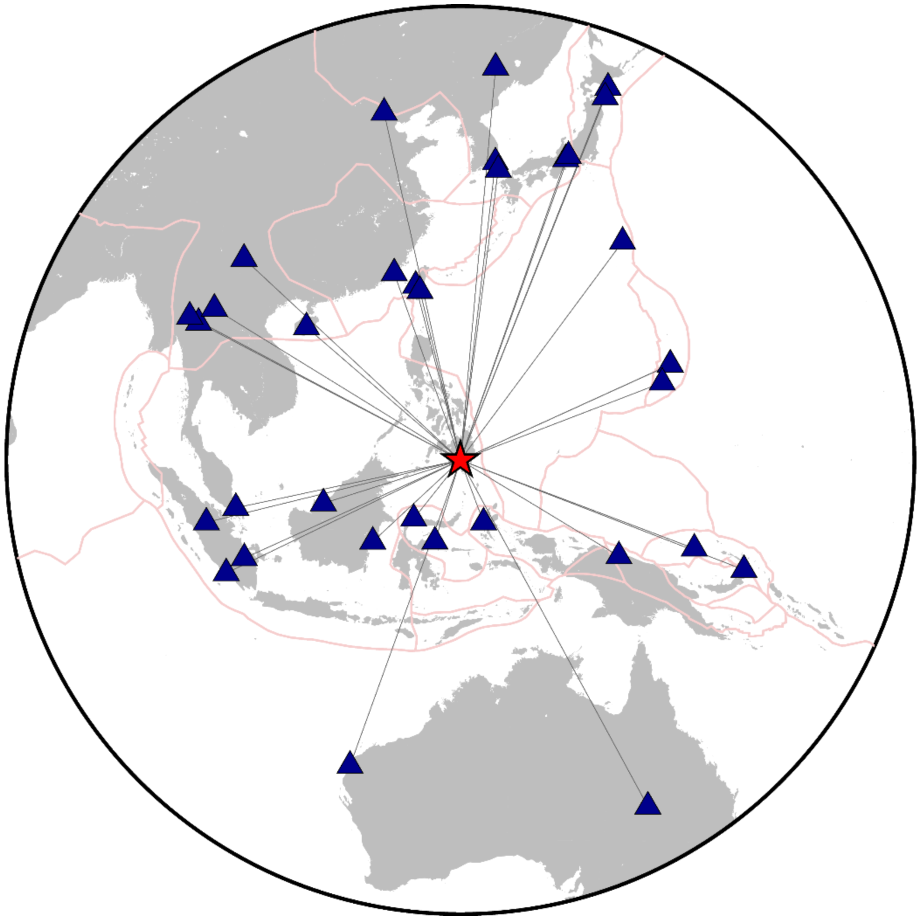

Figure 2. Azimuthal distribution of stations around the Philippine earthquake

In this research, the duration of rupture was estimated using the P-wave by calculating the time delay after the arrival of the P wave of 90% ($T^{0.9}$), 80% ($T^{0.8}$), 50% ($T^{0.5}$), dan 20% ($T^{0.2}$) from its peak value [9-10]. Mathematically, the rupture duration of the P-wave can be determined using the following equation:

$$T_{dur} = (1 - w)T^{0.9} + w\,T^{0.2} \tag{2}$$

$$w = \left[\frac{T^{0.8}+T^{0.5}}{2} - 20s\right]/40s \tag{3}$$

Where w is a constant with a limit of $0 \leq w \leq 1$ and the limit for the rupture duration is

$T^{0.2} \leq T_{dur} \leq T^{0.9}$ [11].

Levenberg-Marquardt Inversion

The rupture duration and seismic station azimuths form a non-linear relationship [12-13]. Therefore, to estimate rupture directivity model and the actual fault plane, it is necessary to do a non-linear inversion. In this case, a local search method commonly applied in geophysical inversion problems is used, namely the Levenberg-Marquardt (LM) method, which uses a linear approach. This algorithm has the advantage that it guarantees good convergence and non-singularity solutions. In addition, this algorithm combines the advantages of the Gauss-

Newton method, which is fast but convergence is uncertain and the Steepest-Descent method, which is very slow but guarantees convergence [13]. Mathematically this method can be written as the following equation:

$$\boldsymbol{m}_{n+1} = \boldsymbol{m}_n + [\boldsymbol{J}_n^T\boldsymbol{J}_n + \lambda\mathbf{I}]^{-1}\boldsymbol{J}_n^T\big(\boldsymbol{d} - \boldsymbol{G}(\boldsymbol{m}_n)\big) \quad (4)$$

where $\mathbf{I}$ denotes the identity matrix, and λ indicates the damping factor or Lagrange multiplier. When the initial iteration increases the value of λ so that the diagonal element becomes dominant, this is the Steepest-Descent method. If the solution improves, λ is reduced, then the LM method approaches the Gauss-Newton method, and the solution is usually accelerated to a local minimum [14-16]. In this case, for forward modelling calculations using (1), which is simplified into the following equation:

$$T_{dur} = a - b * \cos(x - c) \quad (5)$$

In Equation (5) a is L/V_r, b denotes L/V, x indicates φ, and c represent λ^*. Thus, the function $\boldsymbol{G}(\boldsymbol{m}) = T_{dur}\,(x; a, b, c)$ where x denotes the independent variable, while for the model parameter, $\boldsymbol{m} = (a, b, c)$ will be inversed. In Equation (4), the Jacobian matrix has the mathematical equation $\boldsymbol{J} = \partial\boldsymbol{G}(\boldsymbol{m})/\partial\boldsymbol{m}$ which is the partial derivative of the function $\boldsymbol{G}(\boldsymbol{m})$ for each parameter of the $\boldsymbol{m}$ model. Therefore, the representation of the matrix $\boldsymbol{J}$ is the following equation:

$$\boldsymbol{J} = \left[{}^{\partial\boldsymbol{G}(\boldsymbol{m})}/_{\partial\boldsymbol{a}} \;\; {}^{\partial\boldsymbol{G}(\boldsymbol{m})}/_{\partial\boldsymbol{b}} \;\; {}^{\partial\boldsymbol{G}(\boldsymbol{m})}/_{\partial\boldsymbol{c}}\right] \quad (6)$$

In equation (6), each component of the Jacobian matrix is derived based on equation (5) which is then used as input to determine the calculation data $\boldsymbol{d}^{cal}$. Furthermore, we will get the value difference between $\boldsymbol{d}^{obs}$ and $\boldsymbol{d}^{cal}$. If the value of $\Delta\boldsymbol{d}$ is large, then an evaluation of the initial parameter model is carried out to obtain the best solution. Meanwhile, if the value of $\Delta\boldsymbol{d}$ is small, it indicates that the solution has been fulfilled. In order to obtain a suitable solution or there is a match between the calculated data and the observation data, it can be done by minimizing the objective function. In this case, the objective function used is root mean square error (RMSE), which can be written as the following equation:

$$RMSE = \sqrt{\textstyle\sum_{i=1}^{n}(\boldsymbol{d}^{obs} - \boldsymbol{d}^{cal})^2/n} \quad (7)$$

where n is the number of data points, $\boldsymbol{d}^{obs}$ and $\boldsymbol{d}^{cal}$ are observation and calculated data, respectively.

Result and Discussion

The rupture directivity of the Philippines earthquake on 15 December 2019 has determined using teleseismic data with an epicentral distance of about 40° that consist of 35 stations, assuming that the rupture propagates unilaterally [17]. The stations used to calculate the rupture duration are 35 stations that are evenly distributed around the earthquake. The curve fitting results between the rupture duration and azimuth of seismic stations using the LM method can be seen in the Figure 3.

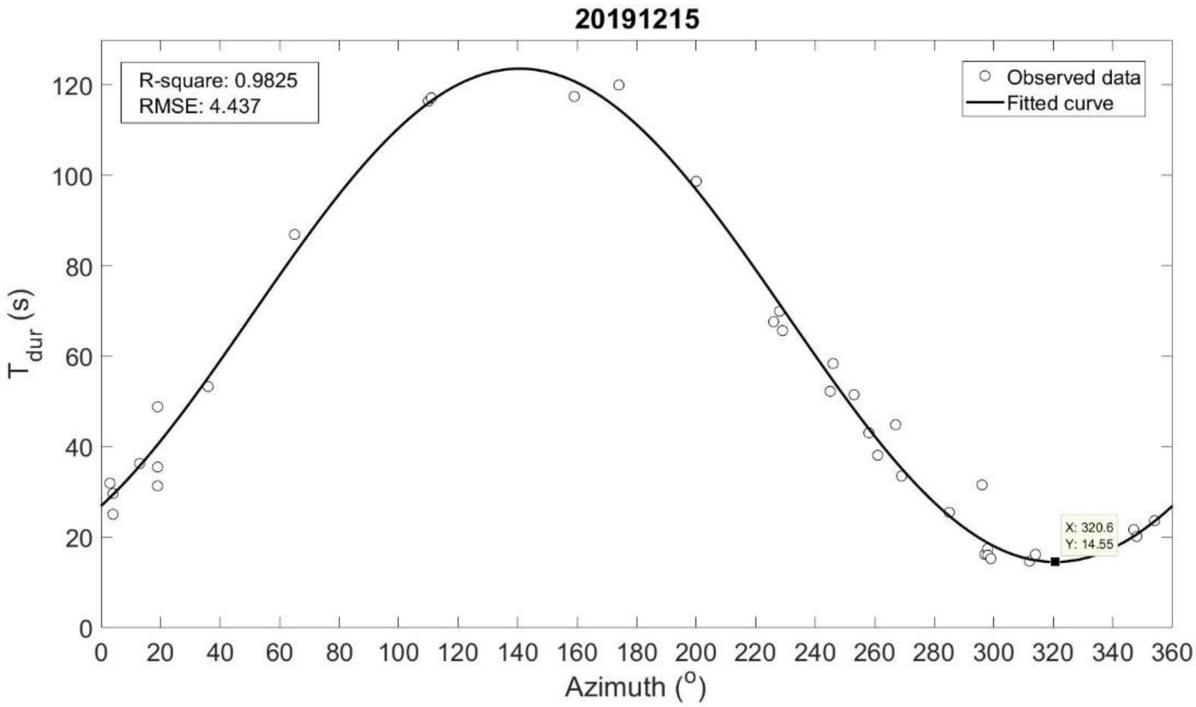

Figure 3. The curve fitting result of rupture duration data ($Tdur$) using LM inversion

Figure 3 describes that the calculation data (black line) and observation data (white circle) of the rupture duration have a good match (best fit) which is indicated by R-squared value of 0.9825 and a Root Mean Squared Error (RMSE) value of 4.437. The initial model used is $a = 1$, $b = 1$, and $c = 300$ with a damping factor (λ) value is 0.01. The best model parameters obtained from the inversion using the LM algorithm are $L/V_r = 69.08 \pm 1.85$, $L/V = 54.53 \pm 5.63$ and $\lambda^* = 320.60 \pm 2.30$ with 12 iterations. This small iteration is caused by the determination of the initial model close to the solution and the observation rupture duration data are well distributed. Based on the inversion model parameters, especially the model parameter λ^*, this earthquake has a rupture propagation direction towards the northwest (NW) from the epicentre at an azimuth of 320.60° $\pm$ 2.30° . The uncertainty value of this parameter is relatively small, so it can be indicated that the resulting model parameter is quite accurate. According to equation 1, the minimum rupture duration value represents the rupture directivity of an earthquake [18]. In this earthquake, the minimum rupture duration was recorded at 320.60° azimuth with a value of 14.55 s. Thus, the rupture directivity model parameter using LM inversion has a good agreement with the results of rupture directivity using the minimum rupture duration value approach. Based on the results of the focal mechanism of the GCMT and USGS, this earthquake has a strike-slip fault type. The rupture directivity can be used to distinguish between the actual fault plane and the auxiliary plane in the case of earthquakes having a strike-slip faults [19]. Comparison table of the earthquake focal mechanism and rupture directivity in this study have presented in Table 1.

The rupture directivity has the same position as the strike of the focal mechanism solution, which is measured from the north direction of the earthquake source. Table 1 indicates, the direction of rupture in the azimuth 320.60° ± 2.30° which corresponds to the strike value of nodal plane 1 (NP1). This indicates the actual fault plane in the Philippines earthquake on 15 December 2019 was NP1.

Table 1. Focal mechanism of the 15 December 2019 earthquake.

Source	Nodal Plane 1			Nodal Plane 2			Rupture Directivity in this study
	S	D	R	S	D	R	
GCMT [20]	320^o	74^o	13^o	226^o	77^o	163^o	
USGS [21]	319^o	82^o	-34^o	54^o	56^o	170^o	320^o $\pm$ 2.30^o
InSAR [4]	320^o	75^o	17^o	-	-	-	

Strike (S), Dip (D), Rake (R)

The rupture propagation process in the earthquake shows that the SE-NW orientation of the fault is part of the Cotabato fault system [22]. Several authors show that the Philippines earthquake on 15 December 2019 was caused by a strike-slip fault whose rupture propagated along the SE-NW direction, which had similar characteristics to the first earthquake of October 16, 2019 [4, 23]. Therefore, it can be concluded that the determination of rupture directivity using the LM inversion method is reliable and has consistent results with previous studies. Determination of the rupture direction using the LM inversion method is not only for specific earthquakes, but this method can be used for all earthquakes, assuming it has a single rupture direction (unilateral). Unilateral rupture usually occurs in large earthquakes (M_w more than 6.5), this is evident in the results of this study which can find a solution for the direction of rupture propagation for the Philippines earthquake M_w 6.8. Thus, this method is advantageous for finding the rupture direction of large earthquakes.

Conclusion

The rupture directivity of the Philippine earthquake on 15 December 2019 was estimated by the rupture duration value of each seismic station. The Levenberg-Marquardt inversion method was used for curve fitting between the calculated and observed rupture duration to obtain the model parameter of rupture directivity. The curve fitting result using the LM method has a good agreement between the observed and the calculated data. The model parameter results show that the rupture propagates in the northwest (NW) direction with azimuth 320.60^o $\pm$ 2.30^o. Based on rupture directivity, this earthquake has an actual fault plane, namely NP1 with a strike/dip/rake value are 321^o /75^o /13^o , respectively (GCMT focal mechanism solution). These results also reveal that the Philippines earthquake on 15 December 2019 has a fault orientation in the SE-NW direction, which is part of the Cotabato fault system.

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