An Alternative Formulation of Physical Laws of Motion

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ARTICLE INFO

Article History:
Received: 22-09-2020
Revised: 26-09-2020
Accepted: 29-09-2020

Keywords:
Newton’s Laws; First Year Physics; Conservation of Momentum


DOI: https://doi.org/10.29303/ipr.v3i3.66

ABSTRACT

The purpose of this paper is to present a logical, consistent and understandable alternative formulation of physical laws of motion. This paper gives also a different view and understanding of physical laws of motion. The central idea of this formulation is the concept of momentum and its conservation. This idea is emphasized in this paper. It is shown that Newton’s laws of motion are given as a consequence of the law of conservation of momentum.

Introduction

Classical mechanics is usually studied at universities by first-year science and engineering students. The lecturers of the subject usually explaining Newton's laws of motion at the beginning of the course after the kinematic motions. This is also adopted by all university physics textbooks such as by Giancoli [1], Halliday et al. [2], Ohanian [3], Serway and Jewett [4] and Urone [5]. This is the common acceptable practice for learning classical physics.

In teaching classical physics courses at high school and university level, previous papers [6-11] reported that students had difficulties understanding Newton's laws, especially the first and third laws. To improve student understanding of the laws of motion, we believe that a new way to approach Newton's laws or an alternative formulation of Newton's laws is needed. Eisenbud [11] reformulated Newton's laws in order to give better clarity of the laws without changing their order, meaning or interpretation. Stocklmayer et. al [13] reported that by changing the order of presentation of Newton's laws, they found that student's
understanding of the laws has improved significantly. Lutz et. Al [14] also reported that by reversing the order of Newton laws gives positive effect to students understanding of the laws.

Besides that, our understanding about our universe has changed dramatically since many discoveries in astronomy and cosmology. The theory of relativity and gravitation have also contributed to this change of our view of the universe [15, 16]. This new view of our universe should be included in teaching classical mechanics. In other words, the laws of universe should be connected to our universe.

The definition or concept of momentum and its conservation are usually presented after explanation of Newton's third law. In this order, it can be thought or believed that the conservation of momentum is a consequence of Newton's third law [3, 4]. This is mostly understood by students and surprisingly by lecturers too. However, when we consider it carefully, we then realize that the conservation of momentum should be the first concept to be understood.

Besides that, Newton's laws seem to be disconnected from each other. Newton's three laws are not related to each other even though all laws use the concept of force. We will show in this paper that using the concept of conservation of momentum gives coherent unifying physical laws.

In this paper, we propose an alternative formulation of physical laws of motion where the concept of momentum becomes the central role. The new physical laws are logical, consistent and easy to understand.

**Alternative Formulation Physical Laws**

Our proposal of physical laws of motion is the following:

**Law 1:**

For an isolated object, the object has a constant momentum. The momentum of the object is proportional to its velocity.

\[ \mathbf{p} = m\mathbf{v} = \text{constant} \quad (1) \]

where \( m \) is the proportionality constant and it is not yet defined here. It will be understood later after the statement of the second law. This first law is equivalent to Newton's first law [3].

This view is connected to the idea of our universe as a whole. To understand further, let us consider an empty universe. Obviously, there is no physical law of motion in this empty universe. We then add an object in this empty universe. We then ask a question about one-object universe: What do physical laws of motion exist in this universe? One possible law is that the velocity of the object is a constant or zero (Newton's first law). The choice between velocity or momentum is a matter of preference. In order to be consistent with the second law of motion, we choose the momentum for the first law. It is also more general to use the
momentum since the constancy of velocity is a special case of the constancy of momentum with one as the proportionality constant.

For one object-universe, we can think of only one law of motion. After we add more objects to one-object universe, we can add other physical laws of motion. We explore this case when we consider second and third laws of motion.

**Law 2:**
*For an isolated system consisted of two or more objects, the total momentum of the system is always constant or conserved.*

The word “isolated” in this law means that there is no interaction between the system and the environment or other objects. This law is actually the same as the first law (Law 1) if we think of the whole system as a single object. This comes as no surprise after knowing the first law. The second law is consistent with the first law.

The total momentum of objects is

\[ \sum p_n = \sum m_n v_n = \text{constant} \quad (2) \]

To understand further, let us consider a system of two interacting point objects (let’s called them Object 1 and Object 2) with initial momentum \( p_{1i} = m_1 v_{1i} \) dan \( p_{2i} = m_2 v_{2i} \) before interaction. After interaction for an interval of time \( \Delta t \), the momenta change to \( p_{1f} = m_1 v_{1f} \) dan \( p_{2f} = m_2 v_{2f} \). The total momentum is conserved. This means that the total initial momentum must be the same as the total final momentum. Therefore,

\[ p_{1i} + p_{2i} = p_{1f} + p_{2f} \quad (3) \]

After simple rearrangement such that terms involving Object 1 on the left-hand side and Object 2 on the right-hand side, we then have

\[ [p_{1f} - p_{1i}] = -[p_{2f} - p_{2i}] \quad (4) \]

or

\[ \Delta p_1 = -\Delta p_2 \quad (5) \]

This equation indicates that the change of the momentum of Object 1 is opposite to the change of the momentum of Object 2. This means also that the momentum is transferred from one object to the other [1, 3].

The proportionality constant \( m_1 \) and \( m_2 \) have not yet been defined. To understand the meaning of \( m \), let us first consider two identical interacting objects. Their proportionality constants must be the same, \( m_1 = m_2 = m \). This is understandable if we consider when they are not interacting. Each object lives in its own universe that is equivalent to each other and
as a consequent the proportionality constant has to be the same since they are identical objects.

We then consider three identical objects. We consider one object (labeled by index 1) interacting with the other two objects (index 2 and 3). The change in momenta during interaction are

\[ m \Delta v_1 = -m \Delta v_2 - m \Delta v_3 \]  
(6)

For a case when \( \Delta v_2 = \Delta v_3 \) or object 2 and object 3 are sticking together, we then have

\[ m \Delta v_1 = -(2m) \Delta v_2 \]  
(7)

This means that for two identical objects merged into one object, the proportionality constant is twice of the original object, that is \( 2m \). We can do similar reasoning for different objects. Therefore, in general the proportionality constant, \( m \), is the sum of the proportionality constants of comprising objects, \( m = \sum_i m' \). This means also that the proportionality constant is equal to the size or amount of matter in the object. This is what we define as the "mass" of the object [17].

Let us consider further Eq. (5). Because Object 1 and Object 2 interacts for an interval of \( \Delta t \), the average rate of momentum transfer is defined as

\[ \frac{\Delta p_1}{\Delta t} = -\frac{\Delta p_2}{\Delta t} \]  
(8)

We define a quantity, known as a force, given by the rate of change of momentum

\[ \bar{F} = \frac{\Delta p}{\Delta t} \]  
(9)

If the time is infinitesimally small, then the force is defined as

\[ F = \lim_{\Delta t \to 0} \frac{\Delta p}{\Delta t} = \frac{dp}{dt} \]  
(10)

From Eq. (8) and the definition of force (Eq. (9) and (10)), we can conclude that

\[ F_1 = -F_2 \]  
(11)

The force acting on Object 1 is the same but opposite direction as the force acting on Object 2. This is the statement of Newton's third law.

Because force is not yet defined, we state it as the third law of motion.
Law 3:
An interacting force on an object is defined as the time rate of change of momentum of the object caused by the force or

$$ F = \frac{dp}{dt} \quad (12) $$

This is the statement of Newton's second law.

Discussion

Three laws as stated previously are developed sequentially in order of importance. The statements of the three laws are not new. There have been presented in textbooks such as Ohanian [3]. Because the concepts of conservation of momentum are usually discussed after statements of Newton's law. The important of momentum has become faded, even though the equivalency of the laws are mentioned in the text. We often find our university students neglect the fundamental of conservation of momentum.

Our alternative laws of motion are all related to the concept of momentum. The first law introduces the definition of momentum and its constancy, the second law introduces the conservation of momentum, the third law relates the definition of rate of change in momentum during interaction. This rate of change is then used to define the force. This presentation of physical laws of motion gives unification of the laws of motion which is easier to understand compared to separate laws as in Newton's laws. Knowing simpler statements of natural laws with one idea in mind, that is momentum, will reduce burden to the students in remembering three laws of Newton.

We can also relate our alternative formulation of physical laws to Newton's laws. The order of Newton's laws are changed where Newton's third law is stated as the second law before the Newton's second law. This indicates that when teaching Newton's laws and to improve student grasp, we need to reorder the presentation of the laws as reported by Stocklmayer et. Al [12].

Beside of reordering Newton's laws, the idea of mass which is the proportionality constant in the definition of momentum is understood by using the second law. The mass of an object is related to the amount of matter in an object. This has been shown directly as the implication of the second law and it cannot be derived by the first law. Because the second law indicates interaction with other object, this implies that the mass of object also related to the interaction of the object with other objects in the universe. This means that the present of other objects in the universe and their interactions are the cause of their masses.

Conclusion

We have presented a logical derivation of physical laws of motion using the constancy or conservation of momentum as the fundamental concept. We then have derived Newton's laws of motion from the conservation of momentum. The mass of an object has been shown as an implication of the conservation of momentum (or second law). The alternative formulation of laws of motion are easier to be grasped by students.
Acknowledgment

We gratefully acknowledge all our students who have participated actively in first year physics class and asked many interesting questions. We would like also to thank Dian W. Kurniawidi for fruitful discussions.

References